

In astrophysics, high energy, and nuclear physics, relativity is central to understanding many observations.

The canonical experiment can be boiled down to the following: Particles are produced (event 1), they decay (randomly), and are detected (event 2) at some location that is not where they are produced.

This "random" decay is a Poisson process and can be modeled continuously using,

$$\frac{dN}{dt} = -\lambda N \quad \text{where } \lambda \text{ is related to the rate of decay.}$$

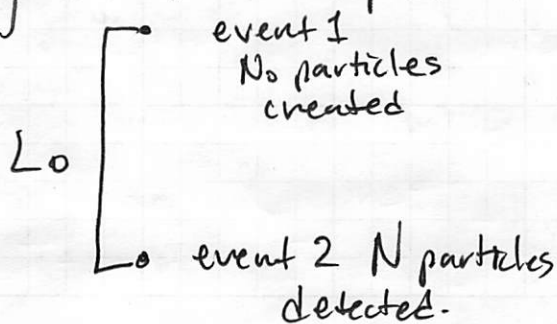
If we start with N_0 particles then,

$$\frac{dN}{N} = -\lambda dt \Rightarrow \ln(N(t)) - \ln(N_0) = -\lambda t$$

so that $\frac{N(t)}{N_0} = \text{fraction of surviving particles} = e^{-\lambda t}$

High Speed Particles $v \sim c$

What does the detector see? Assume that all N_0 particles are generated at one location and they are all on a path to the detector.



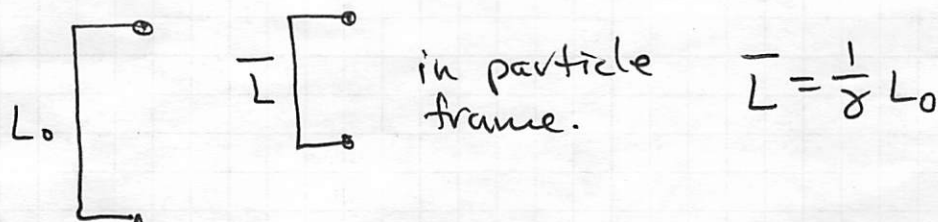
L_0 is the distance in the detector frame.

What fraction of particles does the detector see?

$$\frac{N(t)}{N_0} = e^{-\lambda t} \quad \text{But is } \Delta t = \frac{L_0}{v_{\text{particle}}? \quad ?$$

in the detector frame?

We would underestimate the fraction of particles in this case. Because in the particle frame, the distance travelled is shorter.



but the particles traverse this with a given v_{particle} ,

$$\frac{L_0}{\Delta t} = v_{\text{particle}} = \frac{\bar{L}}{\Delta t_{\text{proper}}}$$

B/c this Δt_{proper} is measured with one clock (namely the one in the rest frame of the particles, it is Δt_{proper}).

thus it's the shortest time in any frame.

$$\Delta t_{\text{proper}} = \frac{\bar{L}}{L_0} \Delta t = \frac{\Delta t}{\gamma}$$

$$\frac{N(t)}{N_0} = e^{-\lambda \Delta t_{\text{proper}}}$$

this fraction is higher than the one in the detector frame, and is in fact the one we get in experiments!