Special Theory of Relativity

the idea of relativity is quite basic:

- All observers in inertial frames agree on the laws of physics.*

* This does not mean that observers will agree on everything or that events occur in the same location or at the same time measured by all observers.

- Moreover, it challenges our usual physical intuition where the ~~things~~ events we observe occur at times that are relatively slow (i.e., the observations we make in our everyday life occur so slowly that the effects of special relativity are unobservable).

- However, when we start to approach speeds (or rates) comparable with c (or rates of c), then what we define as an event, an observation, and time & space become necessarily more complicated.

- We must be very careful with how measurements are made, how events are defined, and even how an observation is thought of.

* Space & time are no longer seperable. *

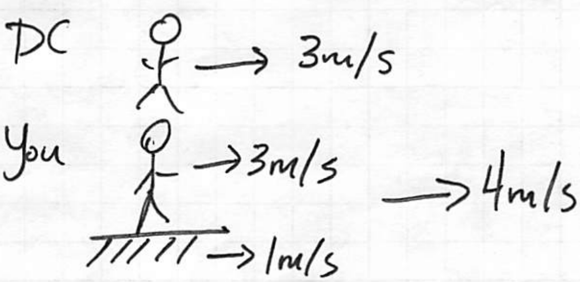
Let's remind ourselves of relativity in the Classical sense (Galilean transformations)

<u>Newtonian</u> <u>Principle</u> <u>of Relativity</u>

I walk along the ground at 3m/s. You are walking on a moving walkway in the same direction. The walkway moves at 1m/s and you walk at 3m/s.

this simple example fits with our everyday experience and it confirms our usual understanding of reference frames and the physics observed in each frame.

<u>Relative</u> <u>to</u> <u>me</u> you move 1m/s (say to the right)



Because we both move at constant speeds, any classical mechanics experiments we do will yield the same outcome.

However, when I try to understand your experiments from my frame, I have to always subtract 1m/s to the right to get your results.
(or add 1m/s to the left)

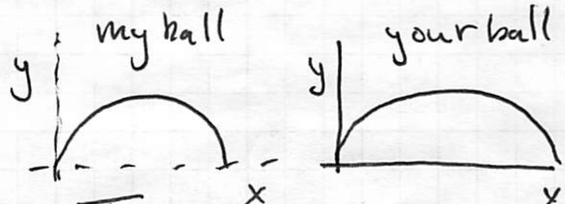
You have to subtract 1m/s to the left (or add 1m/s to the right) to get mine.

Neither of us agree with the person sitting at rest on the bench who always has to ~~add~~ subtract either 3m/s (for me) or 4m/s (for you) to agree that their measurements of our experiments match.

Example: we all 3 toss a ball in the air straight up in our frames.

To each of us, the ball moves straight up and straight down. However, comparing each observation of the other (from our respective frames) means we have to shift (transform) our measurements of other experiments in our own frames.

→ sitting person sees:

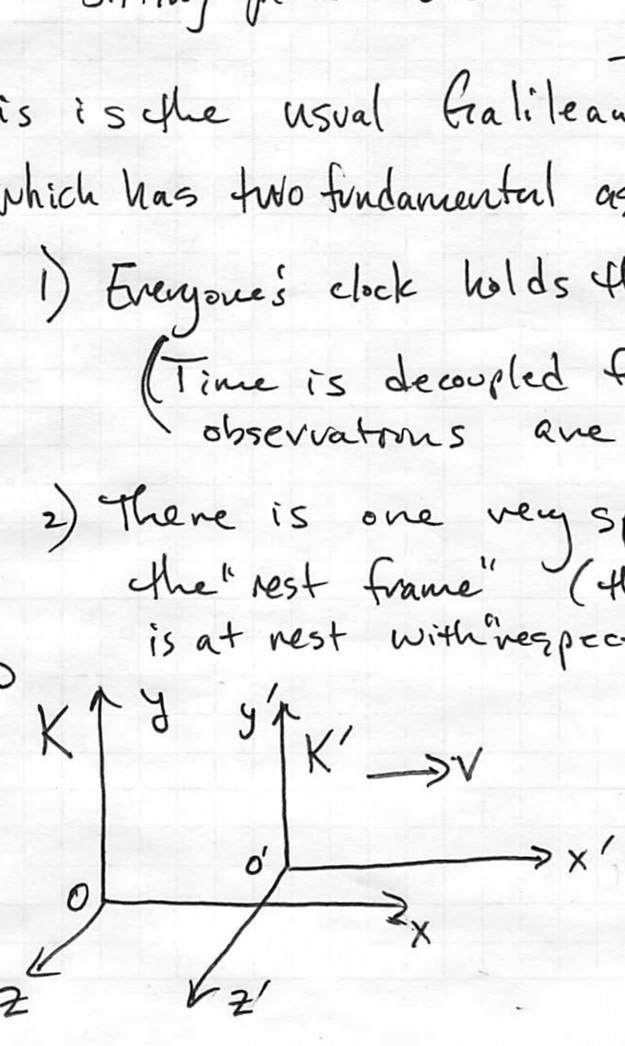


This is the usual Galilean Transformation, which has two fundamental aspects:

1) Everyone's clock holds the same time (Time is decoupled from space, thus observations are basically "seeing")

2) There is one very special frame called the "rest frame" (the sitting person) with is at rest with "respect to the fixed stars"

In 1D



K is the rest frame

K' is the moving frame.

Comparing K and K', which moves at a speed v with respect to K, we find,

$$x' = x - vt \qquad y' = y \qquad z' = z \qquad \underbrace{t' = t}$$

Newton considered time to be absolute so it is decoupled from space!

the inverse transform is quite simple,

$$x = x' + vt \qquad y = y' \qquad z = z' \qquad t = t'$$

Only the x equation changes here in 1D.

[If K' moves with a velocity $\vec{v}$ relative to K then,

$$\vec{r}\,' = \vec{r} - \vec{v}t \qquad t' = t$$

and

$$\vec{r} = \vec{r}\,' + \vec{v}t \qquad t = t'$$]

Much of your every day experience comes from this kind of transformation. One of the major conceptual challenges w/ special relativity is that space & time are now coupled thanks to a <u>constant speed of light in every frame</u>.

What does any of this have to do with E&M?

Maxwell showed that waves in vacuum travel at c.

$$\nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

From this, he (and others) showed that in media there's a well-defined speed in those media for waves.

$$\nabla^2 \vec{E} = \frac{1}{v_0^2}\frac{\partial^2 \vec{E}}{\partial t^2} \qquad v_0 = \frac{1}{\sqrt{\mu \varepsilon}}$$

But what happened when you start to think about how Maxwell's equations account for measurement in different frames?

→ At the time, the presumption was that somehow, the work of Galileo and Newton should translate over. That is, that for a frame moving with relative to a fixed frame that,

$$c \to v \pm c \qquad V_0 \to V_0 \pm V$$

depending on direction of travel.

→ In fact this works for water waves, sound waves, elastic media, etc. etc. ...

☆ It wasn't a crazy idea to think EM waves must have some medium through which the travel, which might have some speed of it's own (and thus suggest some completely fixed frame → "the rest frame" → that all observers could use).

Michelson and Morley set out to find the "ether" and to quantify it's movement. However, even with an ability to detect the result at a resolution ~1000 times higher than the expected signal, they failed.

There experiment was repeated many times in a variety of settings to no avail.

Led to the proliferation of the "ether drag" hypothesis, that Earth somehow dragged the Ether with it.

Enter Einstein...

Einstein's Postulates

frustrated by Maxwell's equations and there seemingly broad applicability and yet simple interpretive failures, Einstein proposed 2 radical Ideas.

Background → Maxwell's Equations suggested that a moving wire will experience an EMF as it enters a magnetic field. this was due to magnetic forces.

However to an observer riding on the wire, there is no velocity, but the EMF is still observed because the magnetic field changes producing an electric field.

in Both case $\mathcal{E} = -d\phi/dt$ but the interpretation of one is that $\vec{F} = q\vec{v}\times\vec{B}$ is responsible while in the other it's, $\oint \vec{E}\cdot d\vec{l} = \frac{-d}{dt}\int \vec{B}\cdot d\vec{A}$!

But these are just frame changes, so what's the deal?

Einstein proposed:

1. The laws of physics apply in all inertial frames
   - → there is no way to detect absolute motion
   - → and there is no preferred inertial frame
2. The speed of light is constant in all inertial frames and that constant is c!
   - → In vacuum, everyone will measure the same c.

Postulate 2 is pretty hard to fathom because it necessarily couples space & time so that everyday concepts like event, observation, and measurement change significantly.

⇒ It's also NOT a philosophical point but consistent with "all" experiments conducted so far.

[A flashlight in a moving train makes a beam that travels at c with respect to both the train and the ground!]

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3 Consequences of Einstein's Postulates

First, we must define an "event."

→ An event is something that happens at one place and time (x, y, z, t).

→ for example, light beam strikes a detector, firecracker explodes, light is lit, light goes out, etc.

① Relativity of Simulaneity – Two events at two different locations observed* as simultaneous in one frame can occur in a different time order in other frames (in either order, in fact!)

[*observed has a special meaning]

② Time Dilation : moving clocks are observed to run slow. The time between two events located at the same place in one frame (known as "proper time") is always shorter than the time observed for those two events in any other moving inertial frame!

(3) Length contraction: Moving objects are observed to be shorter in the direction of motion. The distance between ends of an object at rest in one frame (the "proper length") is always longer than the length measured by finding the distance between the ends as measured simultaneously in any other inertial frame moving parallel to the object's length.

What is an observation?

⇒ observed does not mean "seen" anymore, like with most Galilean situations.

⇒ looking at or taking a picture of an event might distort your view (e.g. your ruler might look longer not shorter) if it's moving towards you.

⇒ You must take into account light travel time.

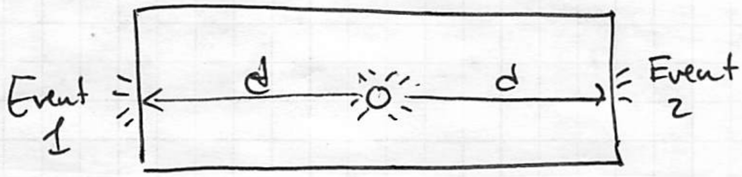
- Start with a "grid" of rulers (at rest) and clocks (at rest) in your frame.
- Synchronize all the clocks and rulers using $\frac{d}{c}$ travel time.

If you look at a faraway clock, you will see an earlier time. But if you observe the clock, it is in exact agreement with your clock. They are in synch!

Simultaneity Example

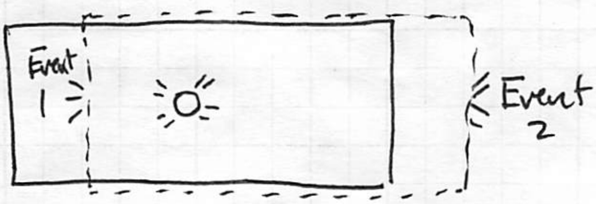
→ Consider a car at rest with a bulb at the center of the car.

A flash goes off and is detected at either end of the car.



Event 1 and 2 (light detection) are simultaneous w/ each other.

→ Consider that car moving to the right (or rather, observed from a left moving frame). The car appears to move to the right.



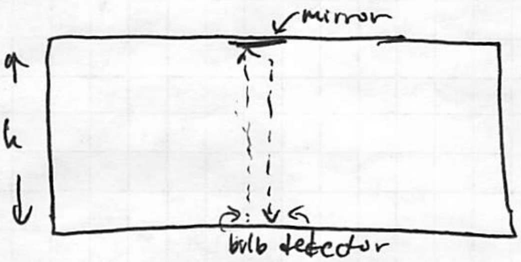
In this left moving frame, event 1 happens first and is not simultaneous with event 2.

The speed of light is c in this moving frame, but the left wall (and thus detector) moves forward to meet the beam, so detector #1 flashes at an earlier time than detector #2.

A third observer in a right moving frame will observe event 2 happening first.

So observers do not have to agree on whether distant events are simultaneous → it's a frame dep. concept.

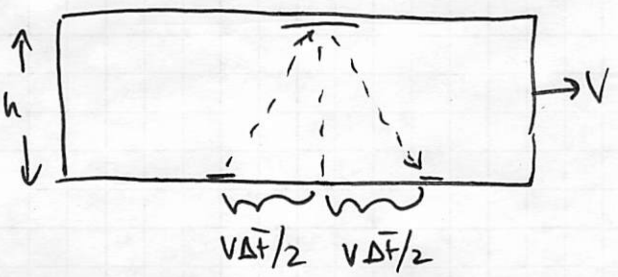
Time Dilation Example



Consider another car at rest. A flash bulb at the bottom flashes, reflects of a mirror, and is detected. Clock at this location reads a time, $\Delta t = 2h/c$

this is the "proper time" between two events occurring in the same location but at different times, measured by one clock (b/c one location!).

$\Rightarrow$ Consider the same experiment observed in a left-moving frame. In this frame the light travels a distance, $d = 2\sqrt{h^2 + \left(\frac{v\Delta \bar{t}}{2}\right)^2}$



the time between the events is

$$\Delta \bar{t} = \frac{\text{distance}}{c} = \frac{2}{c}\sqrt{h^2 + \left(\frac{v\Delta \bar{t}}{2}\right)^2}$$

always.

Notice: these observations would have to be made by two observers with synched clocks b/c the time it takes light to travel is important. You can't just look at the clock from your original location!

We can solve $\Delta \bar{t} = \frac{2}{c}\sqrt{h^2 + \left(\frac{v\Delta \bar{t}}{2}\right)^2}$ $\quad \frac{c^2 \Delta \bar{t}^2}{4} = h^2 + \frac{v^2 \Delta \bar{t}^2}{4}$

$\Rightarrow \frac{\Delta \bar{t}^2}{4}(c^2 - v^2) = h^2$ thus, $\Delta \bar{t} = \frac{2h}{\sqrt{c^2 - v^2}} = \frac{2h}{c}\frac{1}{\sqrt{1 - v^2/c^2}} = \Delta t\, \gamma$

$\gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}}$ is the Lorentz factor, we will come back to this.

So $\Delta \bar{t}$ in a moving frame $> \Delta t_{proper}$ (by γ). $\gamma \geq 1$

$\gamma \simeq 1$ regular distances

$\gamma \to \infty$ ultra relativistic

Lorentz Invariance: From the time dilation example, we can see that different observers might disagree on the time interval between events (we will see that's true for space intervals, too.)

⇒ But we don't disagree on everything. In the last example Δt was the shortest possible time between the 2 events.

⇒ Δt_{proper} (the shortest time) is something any observer can deduce.

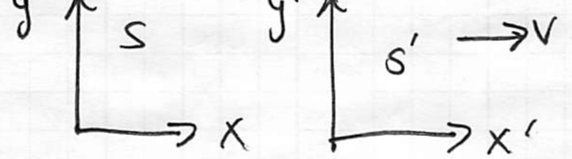
We all agree on it, it's a "Lorentz Invariant" quantity. any $\underline{\Delta t} = \underline{\gamma \Delta t_{proper}}$

⇒ Later we will find other invariants

⇒ they are very useful

Length Contraction perpendicular to relative motion

⇒ There will not be any disagreement among observers regarding length measurements perpendicular to the motion.

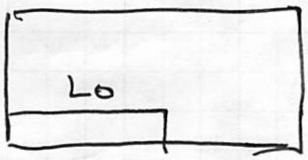


Measurements of y and y' will agree regardless of v.

So we only have to worry about measurements of length in the direction parallel to the motion.

Length Contraction parallel to motion

Consider a car at rest, with a ruler at rest.



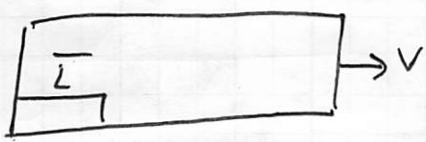
"proper length"

You measure $L_0 = \Delta x$ at one instant (say at $t=0$). This is the proper length of the ruler.

The length of the object at rest.

(Note that "at an instant" doesn't really matter in this frame, since Δx is the same at all times)

Now consider that same ruler observed from a left moving frame. We need to measure $\bar{L} = \Delta \bar{x}$ by locating $\bar{x}_{right}$ and $\bar{x}_{left}$ in this frame at one instant in time.



Because observers in other frames disagree on simultaneity, if I watch the person in the "rest frame" measure the ruler end, I'd say that the measurement was wrong because the right end was measured before the left end!

The result we obtain will be $\bar{L} = \frac{1}{\gamma} L_0$

the moving ruler is observed to be shorter than the rest ruler.

* This is more subtle than simple time dialation. These "tricks" and thought experiments illustrate the weirdness of special relativity but don't give us systematic thinking tools. That will come later $\Rightarrow$ Lorentz Transformations

The "trick" method to do this involves observations at the same location at different times (one observer)

Watch the ruler move past me at speed v,

$\bar{x}=0$, right end passes at $\bar{t}_1$

$\bar{x}=0$, left end passes at $\bar{t}_2$

I would conclude that $\bar{L} = v\Delta\bar{t} = v(\bar{t}_2 - \bar{t}_1)$

I used one clock so $\bar{t}_2 - \bar{t}_1$ is the proper time between the two events

event 1 : right end passes $\bar{x}=0$

event 2 : left end passes $\bar{x}=0$

In another frame, say the rest frame of the ruler

$$\Delta t_{\text{between those 2 events}} = \gamma \Delta t_{proper}$$

one clock ; one location gives us the proper time.

In the rest frame, the observer still have a ruler of length L_0 at rest. the origin of $\bar{S}$ will move to the left in this frame and the right end passes it at t_1 and the left end at t_2. these 2 events are at opposite ends of the ruler, L_0 apart, at a time $\Delta t = t_2 - t_1$ apart.

In this frame $L_0 = v\Delta t$ thus, $\frac{L_0}{\Delta t} = v = \frac{\bar{L}}{\Delta t_{proper}}$

With $\Delta t_{prop} = \frac{1}{\gamma}\Delta t$ $\bar{L} = L_0 \frac{\Delta t_{proper}}{\Delta t} = \frac{1}{\gamma} L_0$ $\bar{L}$ is shorter