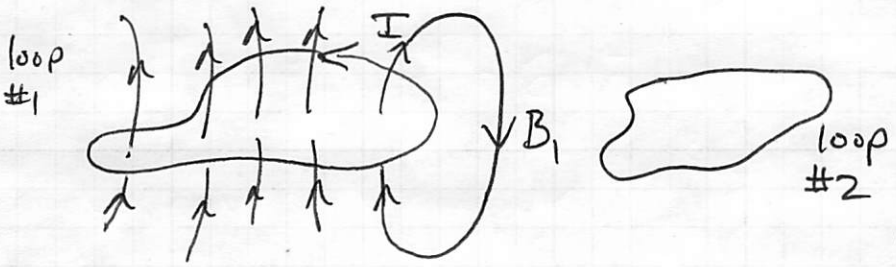
Application of Faraday's Law: Inductance.

Our study of circuits (coming soon) will require that we understand how Faraday's Law comes into situations with multiple loops.

Canonical Conceptual Example



Current flow in loop 1 creates a B-field (B_1) everywhere.

there's a flux ϕ_2 through loop 2.

If I_1 changes then ϕ_2 changes, this induces an EMF in loop 2 (wireless communication)

In principle we could use Biot-Savart to find the field near loop 2,

$$\vec{B}_1(\vec{r}) = \frac{\mu_0}{4\pi} \oint_{\text{loop 1}} I_1 \frac{d\vec{l}_1 \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

where $\vec{\mathscr{r}}$ is the usual separation vector.

This could be a very nasty situation in practice (maybe we could use a computer ??)

But note that $\vec{B}_1 \propto I_1$ so we find that the flux through loop 2 is as well,

$$\underset{\text{due to loop 1}}{\phi_2} = \iint_{\text{loop 2}} \underset{\text{from loop 1}}{\vec{B}_1} \cdot \underset{\text{area is loop 2}}{d\vec{a}_2}$$

So that Φ_2 is,

$$\Phi_2 = \iint\limits_{\text{loop }2} \frac{\mu_0}{4\pi} \oint\limits_{\text{loop }1} I_1 \frac{d\vec{l}_1 \times \hat{\mathscr{r}}}{\mathscr{r}^2} \cdot d\vec{a}_2 \quad \text{Yuck!}$$

But $\Phi_2 \propto I_1$ so we can define a proportionality constant M_{21} which is,

$$\Phi_{2\text{ due to }1} \equiv M_{21} I_1$$

M_{21} is the "mutual inductance." (We will get to its physical meaning soon.)

so if I_1 changes $\Rightarrow$ Φ_2 changes $\Rightarrow$ $\mathcal{E}_2 = -\frac{d\Phi_2}{dt}$ will be present.

thus changing the current in loop 1 drives a current in loop 2.

$$\boxed{\mathcal{E}_2 = -\frac{d\Phi_2}{dt} = -M_{21}\frac{dI_1}{dt}}$$

M_{21} might be hard to compute but if the loops are fixed in size, shape, & orientation M_{21} is constant!

$$M_{21} = \iint\limits_{\text{loop }2} \frac{\vec{B}_1 \cdot d\vec{a}_2}{I_1} = \iint\limits_{\text{loop }2} \frac{\mu_0}{4\pi} \oint\limits_{\text{loop }1} \frac{d\vec{l}_1 \times \hat{\mathscr{r}}}{\mathscr{r}^2} \cdot d\vec{a}_2$$

It's nasty looking, but only depends on geometry (size, shape, orientation of loops).

It's also measurable in the lab using the boxed formula.

We won't do it here, but you can use Stoke's theorem (see Griffiths 7.2.3) to obtain the Neumann formula for M_{21},

$$\boxed{M_{21} = \frac{\mu_0}{4\pi} \oint_{\text{loop } 2} \oint_{\text{loop } 1} \frac{d\vec{l}_1 \cdot d\vec{l}_2}{\imath}}$$

It's still purely geometrical (and not fun to compute), but...

it highlights that swapping 1 and 2 make no difference in the calculation so,

$M_{21} = M_{12}$ this is the origin of the term mutual inductance.

$$M \equiv M_{12} = M_{21}$$

Running I_1 in loop 1 induces $\Phi_2 = M_{21} I_1$

Running I_2 in loop 2 induces $\Phi_1 = M_{12} I_2$

if $M_{21} = M_{12}$ then a 1 amp current in either loop induces the same flux in the other loop.

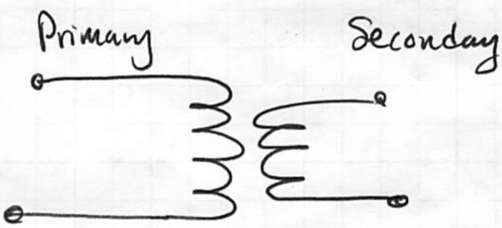
- So practical matter,

  if $\vec{B}$ from loop 1 is easy to find, determine M_{21}

  if $\vec{B}$ from loop 2 is easy to find, determine M_{12}

Either way, $M = M_{12} = M_{21}$,

$$\mathcal{E}_2 = -M \frac{dI_1}{dt}$$



Transformers use this: but you need AC current (d/dt needed!)

Self Inductance

Changing I_1 changes ϕ_1 because there is a magnetic field $\vec{B}_1$ generated by I_1 through A_1 (the loop we are considering).

So this change can produce an EMF
→ termed a Back EMF for reasons that will become readily apparent.

Formally,

$$\phi_1 = M_{11} I_1$$

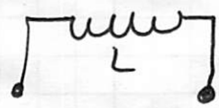
↳ self-inductance, L.

so,

$$\mathcal{E}_1 = -\frac{d\phi_1}{dt} = -L\frac{dI_1}{dt}$$ By definition $L > 0$.

↳ the induced EMF fights the change, hence the term "Back EMF."

Big L helps damp out high frequency noise.
Why? high f means large d/dt
So a large Back EMF is generated to "fight the change."
→ result is that those signals get smoothed out. (We will study these circuits soon.)

In a circuit diagram, 

inductor

units: $[L] = \dfrac{[\mathcal{E}]}{[dI/dt]} = \dfrac{V}{A/s} \equiv H$ for Henry

Also, since $\phi_1 = LI$, $[\phi] = [B \cdot \text{Area}] = [T m^2] \equiv$ Weber, W

A henry is a weber/amp. $1\,H = 1\,W/A$

$[\mu_0] = H/m$ $\quad \mu_0 = 4\pi \cdot 10^{7}\ H/m$