What if there's a capacitor? Can we use the same tools?

 $V = \frac{Q}{C} \Rightarrow \frac{dV}{dt} = \frac{I}{C}$ or that $I = C\frac{dV}{dt}$

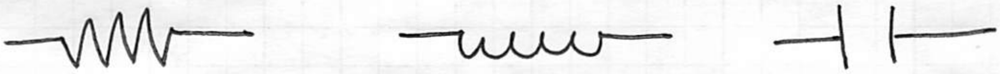
So if the driver is $V(t) = \tilde{V}e^{i\omega t}$ then,

$$I = Ci\omega\tilde{V}e^{i\omega t} = i\omega C V$$

This looks like $V = I\text{"}R\text{"}$ but this time the impedance is $Z_C = \frac{1}{i\omega C} = \frac{-i}{\omega C}$ (we use Z to indicate complex impedance)

Summary

$Z = R$ $\qquad$ $Z = i\omega L$ $\qquad$ $Z = -i/\omega C$



R $\qquad$ L $\qquad$ C

General Result! you can treat each passive element like a simple resistor with the impedances given above. You can constuct Z_{eff} using the standard "rules" for resistors,

In series,

$$Z_{eff} = Z_1 + Z_2 + Z_3 + \ldots$$

In parallel,

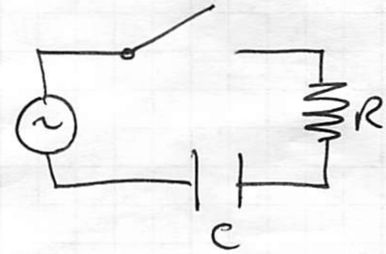
$$\frac{1}{Z_{eff}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \ldots$$

Can apply usual Kirchoff's Laws to them as well.

Example! An RC Circuit

Turn it on @ $t=0$

with $V(t) = V_0 \cos\omega t$



think of this as two Z's,

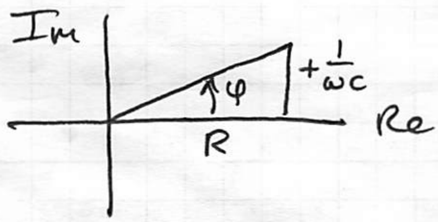
$$Z_{tot} = Z_R + Z_C = R - i/\omega C$$

with, $\tilde{V} = \tilde{I} Z \Rightarrow \tilde{I} = \dfrac{\tilde{V}}{R - i/\omega C}$ and $I_{real} = V_0 \, Re\left(\dfrac{e^{i\omega t}}{R - i/\omega C}\right)$

the first part uses this method again,

$$I = V_0 \, Re\left(\frac{e^{i\omega t}}{R - i/\omega C} \; \frac{R + i/\omega C}{R + i/\omega C}\right) = \frac{V_0}{R^2 + \frac{1}{\omega^2 C^2}} Re\left(e^{i\omega t}\left(R + \frac{i}{\omega C}\right)\right)$$

Use the second method, draw a picture,



$$\varphi = \tan^{-1}\left(\frac{1}{\omega C R}\right)$$

and $$e^{i\varphi} = \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} + \frac{i/\omega C}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

So the particular solution is,

$$I_p = \frac{V_0}{\sqrt{R^2 + 1/\omega^2 C^2}} \cos(\omega t + \varphi)$$

We still need to solve the homogeneous equation,

$$0 = IR + Q/C \Rightarrow \frac{dI}{dt} R = -\frac{I}{C}$$

take time derivative ($dQ/dt = I$)

So, $$\frac{dI}{I} = -\frac{1}{RC} dt \Rightarrow I_H = I_0 e^{-t/RC}$$

So $$I_{tot} = \underbrace{\frac{V_0}{\sqrt{R^2 + 1/\omega^2 C^2}} \cos(\omega t + \varphi)}_{\text{steady solution}} + \underbrace{I_0 e^{-t/RC}}_{\text{transient dies off}}$$

I_0: TBD by initial conditions.

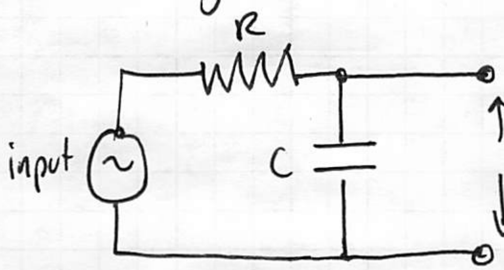
Initial Conditions: ΔV_{cap} cannot suddenly change.
If it was zero before then just after $V_0 = IR$ b/c $\Delta V_{cap} = 0$ for just a quick moment. So $I(t=0) = V_0/R$.

So, this gives,

$$I(t) = \frac{V_0}{\sqrt{R^2 + 1/\omega^2 C^2}} \cos(\omega t + \varphi) + \underbrace{\left(\frac{V_0}{R} - \frac{V_0 \cos\varphi}{\sqrt{R^2 + 1/\omega^2 C^2}}\right)}_{\text{picked so } @ t=0 \ I = V_0/R.} e^{-t/RC}$$

- time constant is "RC"
- After waiting several RC times the circuit oscillates with driver frequency, ω
- If $\omega \to 0$, $I_{\text{long term}} \to 0$, the capacitor blocks steady current.
- if $\omega \to \infty$, $I_{\text{long term}} \to \frac{V_0}{R}\cos(\omega t)$ like the capacitor isn't there.

It's often useful to put this setup in a circuit where you can read out a voltage (as a signal)



We know I in this circuit and V_{out} is,

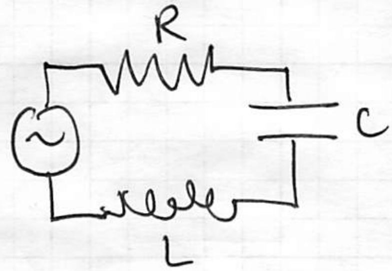
$$V_{out} = I Z_C = I(-i/\omega C)$$

$$V_{out} = \frac{V_{in}}{Z_{tot}} Z_C = \frac{V_{in}}{R - \frac{i}{\omega C}}\left(-i/\omega C\right) = V_{in}\left(\frac{1}{1 + i\omega RC}\right)$$

with $\omega \to 0$, $V_{out} = V_{in}$, cap does nothing

with $\omega \to \infty$, $V_{out} \to 0$, "low pass filter"
Allows low frequencies to pass; suppresses high frequency.

with the phasor method & impedance, any circuit is basically a 184 circuit.



just use $Z = R + i\omega L - \frac{i}{\omega C}$

and then $\tilde{V} = \tilde{I}\tilde{Z}$