Because of the form Maxwell's equations take when $\rho = 0$, we can develop an analogy between Ampere's Law & Faraday's Law.

$$\nabla \cdot \vec{B} = 0 \longleftrightarrow \nabla \cdot \vec{E} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} \longleftrightarrow \nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$$

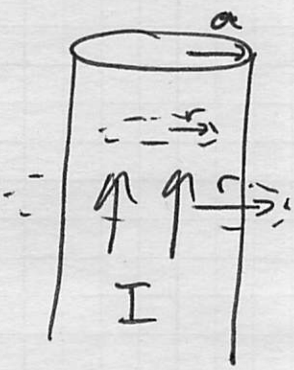
so that,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{J} \cdot d\vec{A} = \mu_0 I_{enc} \longleftrightarrow \oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$$

Recall that the contribution to this integral is only from non Coulombic sources.

Ampere's Example

Remember that we computed the magnetic field inside and outside of a thick wire.



outside: $r > a$ $\qquad \vec{B} = B(r)\hat{\varphi}$ by symmetry

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$\Rightarrow B 2\pi r = \mu_0 I \qquad \vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$

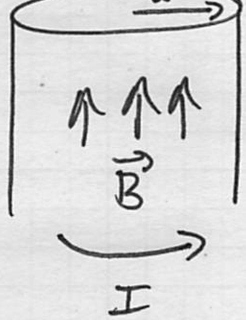
inside: $r < a$ $\qquad J$ is uniform: $J = \frac{I}{\pi a^2}$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$\Rightarrow B 2\pi r = \mu_0 \iint \vec{J} \cdot d\vec{A} = \mu_0 J \pi r^2 = \frac{\mu_0 I \pi r^2}{\pi a^2}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi a^2} r \hat{\varphi}$$

We can use this analogy to determine the electric field around a solenoid,



If it has n turns / length then $\vec{B} = \begin{cases} \mu_0 n I \hat{z} & \text{inside } r<a \\ 0 & \text{outside } r>a \end{cases}$

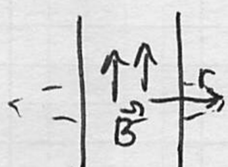
this magnetic field looks precisely like what we had for $\vec{J}$ in the previous example.

If the current changes with time $(I = I(t))$, then so does the magnetic field $(\vec{B} = \vec{B}(t))$.

Farday's Law says,

$$\oint \vec{E} \cdot d\vec{l} = \frac{-d\Phi_{\text{thru loop}}}{dt}$$

We expect (as before) $\vec{E} = E(r)\hat{\varphi}$ so we can draw a Faraday Loop, $(r<a)$



$$\Phi_B = \iint \vec{B} \cdot d\vec{A} = B\pi r^2$$
$$= \mu_0 n I \pi r^2$$

so that $\dfrac{d\Phi}{dt} = \mu_0 n \dfrac{dI}{dt} \pi r^2$

$\oint \vec{E} \cdot d\vec{l} = E 2\pi r$ so that,

$$\vec{E} = -\frac{1}{2\pi r} \mu_0 n \frac{dI}{dt} \pi r^2 \hat{\varphi}$$

$$\vec{E} = -\frac{\mu_0 n}{2} \frac{dI}{dt} r \hat{\varphi} \quad \text{inside}$$

$I\uparrow$ $\vec{E}$ goes ↻
$I\downarrow$ $\vec{E}$ goes ↺
Always "fight the change"

If you are outside the solenoid, we can use the same logic with $\Phi_{enclosed}$ stopping at $r=a$,

$$\oint \vec{E}\cdot d\vec{l} = -d\Phi/dt$$

$$E\,2\pi r = -\frac{d}{dt}\left(\mu_0 n I \pi a^2\right)$$

Φ is zero outside the solenoid b/c $B=0$.

$$\vec{E} = -\mu_0 n \frac{dI}{dt}\frac{\pi a^2}{2\pi r}\hat{\varphi} = -\mu_0 n \frac{dI}{dt}\frac{a^2}{2r}\hat{\varphi}$$

same direction as the previous result.

But notice! $\vec{B}=0$ out there and yet $\vec{E}$ exists throughout space.

this is interesting. we can have a localized source and yet generate something that lives throughout space. (Similar to $\rho \to \vec{E}$ & $\vec{J}\to\vec{B}$)

<u>Comments on the Curl</u>

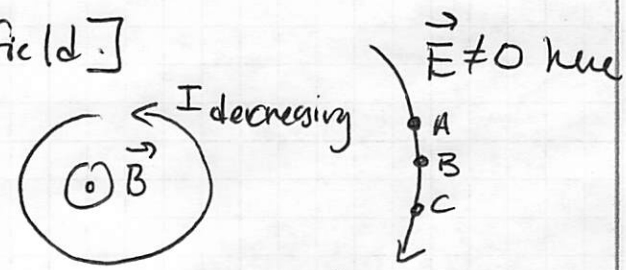
outside the field is $\vec{E} \propto \frac{1}{r}\hat{\varphi}$.

this field circles around, but it has no curl!

??? $\quad \nabla\times\vec{E} = -d\vec{B}/dt = 0$ out here!

Just like $\vec{B}\propto\frac{1}{r}\hat{\varphi}$ outside a wire where $\vec{J}=0$.

[$1/r\,\hat{\varphi}$ is a very special field]



But here's a problem,

with this $V_A > V_B > V_C \longrightarrow$

so that $V_A > V_A$ if we go around!

V is no longer well defined it's path dependent. $\nabla\times\vec{E}\neq 0$ everywhere.

this is why we talk about EMF & not ΔV

$$\oint \vec{E} \cdot d\vec{l} = 0 \quad \text{so that} \longrightarrow \int_A^B \vec{E} \cdot d\vec{l} = V_A - V_B$$

if the path contains no changing flux

is well defined

But if our path contains changing flux then,

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi}{dt} = \mathcal{E}MF$$

We can still compute $\int_A^B \vec{E} \cdot d\vec{l}$ if the path is defined, but this Calculation is not path independent.

Voltage/Potential lose some of their meaning.