Boundary Conditions

- Electrostatics is focused on determining V (or $\vec{E}$) from charges that are configured in some space. This includes both the location and shapes of the charges, whether they exist on a ~~capacitor~~ conductor or in an insulator, et cetera.

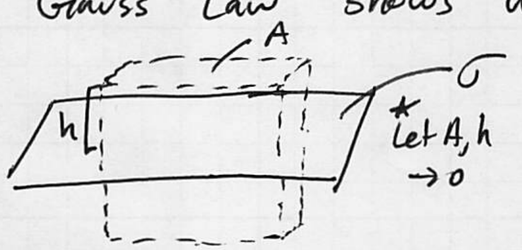
- The field is determined by what is at the boundaries and we will often talk about "boundary conditions", which establish $\vec{E}$ + V

This might be a bit vague now, but consider this:

- At physical boundaries, for example, a sheet with charges:
  - The electric field, $\vec{E}$, can "jump"; it can be discontinuous. Why?
    - Because charges create fields!
  - But the electric potential, V, is always continuous, which can be very useful.

How does $\vec{E}$ jump?

Gauss' Law shows us how (by an amount σ/ϵ_0)



A small Gaussian "pill box" around a sheet of charge with σ surface charge.

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

← the only flux contributions are due to the top + bottom of the box.

$$\oint \vec{E}\cdot d\vec{A} = \vec{E}\cdot\hat{n}_{above} A - \vec{E}\cdot\hat{n}_{below} A = \frac{\sigma A}{\epsilon_0}$$

+ Thus the change in $\vec{E}\cdot\hat{n}$ is σ/ϵ_0 or the amount by which the perpendicular component of $\{\vec{E}\}$ changes is σ/ϵ_0

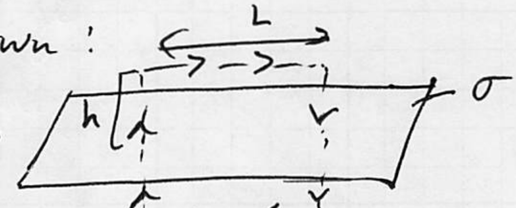
Can we say anything about $\vec{E}$ parallel to the sheet?

Use $\nabla\times\vec{E} = 0$, recall Stokes Law,

$$\int_S \nabla\times\vec{E}\cdot d\vec{A} = 0 = \oint \vec{E}\cdot d\vec{l} = 0$$

So in electrostatics, around any loop the line integral of $\vec{E}$, is zero!

Consider the same sheet of charge and the loop shown:



*Let $h \to 0$

Note you take vanishingly small h to isolate $\vec{E}_{\parallel}$!

The only contributions to $\oint \vec{E}\cdot d\vec{l}$ possibly come from above and below the sheet.*

$$\oint \vec{E}\cdot d\vec{l} = E_{\parallel}^{above}\cdot L - E_{\parallel}^{below}\cdot L = 0$$

so, $\left(E_{\parallel}^{above} - E_{\parallel}^{below}\right) L = 0$ so,

E parallel to the sheet is continuous!

So we have two well established Boundary conditions for $\vec{E}$

① $\Delta\vec{E}\cdot\hat{n} = \sigma/\epsilon_0$ & ② $\Delta E_{\parallel} = 0$

Because $\vec{E} = -\nabla V$ we can rewrite these in terms of V. We'll do that later!

- Finally, we already learned about a boundary condition for V. When we set $V=0$ somewhere, we a saying what the "boundary" value of V is at a location.

- Recall that we typically set this to be $V(r\to\infty)=0$.

  But we don't have to pick $r\to\infty$ for this; we are relatively free to choose as $\vec{E} = -\nabla V$ and any constant will vanish.

<u>Clicker Question</u>: move $V=0$; what happens?