PHY 481
Fall 2019
October 2, 2019

Name: ________________________

Exam #1
Time Limit: 120 minutes

Answer the questions in the spaces provided on the question sheets, **making sure to include units**. If you run out of room for an answer, continue on the pages marked **Extra Work**, but indicate that you have done so.

Your answers should include explanations where necessary (or requested) as well as appropriate units and labels (as needed). Write legibly – If we can't read it, we can't grade it. If you have a question, ask your instructor not your classmate.

Problem	Points	Score
1	20	
2	25	
3	26	
4	29	
Total:	100	

This quiz is to be completed alone with the aid of a single 8.5X11 sheet of paper with your own notes. We have also provided a formula sheet for you.

By signing below, you are agreeing that you have not received unauthorized assistance during this exam, which includes but is not limited to additional crib sheets & note cards, textbooks, course notes, and/or other stored formulas.

Spartan Academic Pledge

As a Spartan, I will strive to uphold values of the highest ethical standard. I will practice honesty in my work, foster honesty in my peers, and take pride in knowing that honor is worth more than grades. I will carry these values beyond my time as a student at Michigan State University, continuing the endeavor to build personal integrity in all that I do.

Signature: ____

Which of the Harry Potter houses would you be sorted into? ____________________

Blank Page

Useful formulas:

$\nabla \cdot \frac{\hat{r}}{r^2} = 4\pi\delta(\mathbf{r})$ $\quad$ $\nabla \cdot \frac{\hat{r}}{r} = \frac{1}{r^2}$ $\quad$ $\nabla \cdot (f\mathbf{A}) = \nabla f \cdot \mathbf{A} + f\nabla \cdot \mathbf{A}$

$f(x) \approx f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2 + \ldots$ $\quad$ $f(x) \approx f(0) + f'(0)(x) + \frac{1}{2}f''(0)(x)^2 + \ldots$

$(1 \pm \epsilon)^n \approx 1 \pm n\epsilon + \ldots$ $\quad$ $\sin\epsilon \approx \epsilon - \ldots$ $\quad$ $\cos\epsilon \approx 1 - \epsilon^2/2 + \ldots$ $\quad$ $\ln(1 + \epsilon) \approx \epsilon - \ldots$

$\ln(a) + \ln(b) = ln(ab)$ $\quad$ $\ln(a) - \ln(b) = \ln(a/b)$

$$\int \frac{dx}{x} = \ln(x) + C \qquad \int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \log\left(\sqrt{x^2 + a^2} + x\right) + C \qquad \int \frac{x}{\sqrt{x^2 + a^2}} dx = \sqrt{x^2 + a^2} + C$$

$$\int \frac{1}{(x^2 + a^2)^{3/2}} dx = \frac{x}{a^2\sqrt{x^2 + a^2}} + C \qquad \int \frac{x}{(x^2 + a^2)^{3/2}} dx = -\frac{1}{\sqrt{x^2 + a^2}} + C$$

$\int \cos x \, dx = sinx + C$ $\quad$ $\int \sin x \, dx = -cosx + C$ $\quad$ $\frac{d}{dx}\cos x = -\sin x$ $\quad$ $\frac{d}{dx}\sin x = \cos x$

$C_{circle} = 2\pi r$ $\quad$ $A_{circle} = \pi r^2$ $\quad$ $SA_{sphere} = 4\pi r^2$

If you need some other integral, just ask!

Blank Page

1. Here's a few questions about delta functions. Make sure your sketches are clear (and label anything you think is important or unclear!)

   (a) (10 points) Sketch the charge distribution $\rho(r, \theta, \phi) = \xi\delta(r - a)$. Think about which coordinate system is being used. Explain where the charge is located.

   (b) (5 points) What are the units of ξ?

   (c) (5 points) Consider an infinitely long cylinder of radius b that carries a constant surface charge density of σ. Write this surface charge density as a volume charge density, $\rho(s, \theta, z)$.

2. You are writing a piece of Python code to model the electric field of a charged rod. In this problem, you will discuss the algorithm and write the statements necessary to model the field.

   (a) (10 points) In your own words or with diagrams/flowcharts, describe the algorithm that calculates the electric field of an extended object at some given location. *Make sure there's enough detail in your description that it could be followed easily.*

   (b) (5 points) Explain how your diagrams/flowchart above is similar to the analytical expression for Coulomb's Law:

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\iiint_V \frac{\rho(\mathbf{r}')(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}d\tau'$$

(c) (10 points) Below is a block of code (a function) that could be used to implement the algorithm you discussed in part (a). Fill in the missing steps to complete the code.
You may assume that numpy has been imported.
If you cannot remember specific syntax, that's ok.

```
def ComputeE(rObs, Nchunks = 20, Q = 0.1e-6, L = .2):

    k = 9e9                               ## Electric constant

    chargeOfChunk = Q/Nchunks             ## Charge of a single chunk

    ## Location of the ends of the rod
    lineStartX = -L/2
    lineEndX = L/2

    ## Create locations for the chunks that are equally spaces
    xLocations = np.linspace(lineStartX, lineEndX, Nchunks)
    yLocations = np.zeros(Nchunks)

    ## Stack the locations so that they are regular (x,y) ordered pairs
    chunkLocations = np.stack((xLocations, yLocations), axis=1)


    dE = np.array([0, 0])
    E = np.array([0, 0])

    ## ENTER YOUR CALCULATIONS HERE
    for thisLocation in chunkLocations:






    return E
```

3. Consider a sphere of radius a centered on the origin. The sphere carries a volume charge throughout.

   The charge distribution inside the sphere varies with radius according to $\rho(r) = -\rho_0 \left(\frac{2r}{a} - 1 \right)$.

   (a) (4 points) Sketch a graph of the charge distribution (ρ vs r). Make sure to show what happens for $r > a$. Make sure to label your graph.

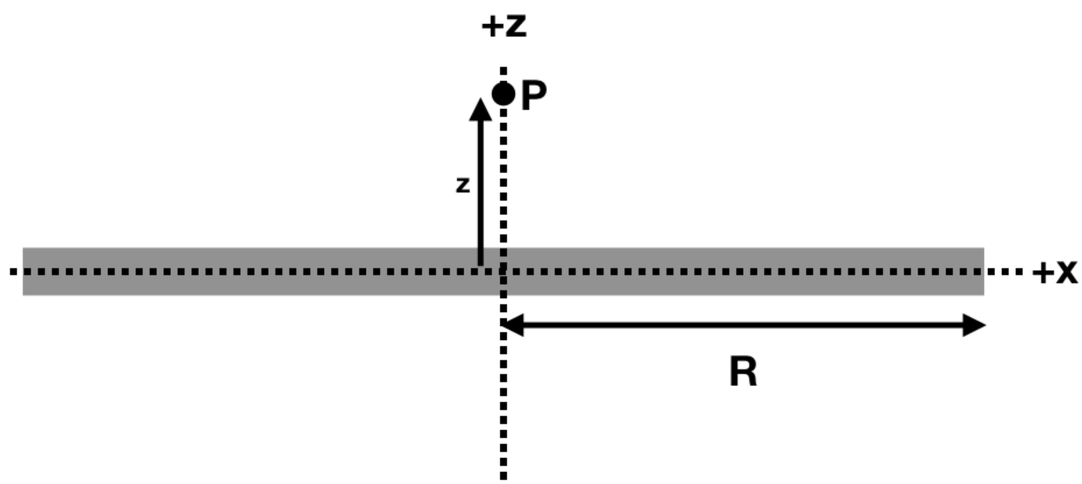
   (b) (6 points) Find the magnitude of the electric field outside the sphere ($r > a$). Which way does the electric field point?

(c) (6 points) Find the electric field inside the sphere ($r < a$).

(d) (6 points) Sketch the r component of the electric field as a function of distance from the origin ($E(r)$ vs r). Make sure to point out any important features in the sketch and label your axes. *Your graph is correct if it is consistent with parts (b) and (c).*

(e) (4 points) Did you use Coulomb's Law or Gauss's Law to solve parts (b) and (c)? Why? Would your approach change if the charge density varied with θ?

4. Consider a thin disk of radius R with a charge density that varies radially and azimuthally, $\sigma(r, \phi)$. It's center is located at the origin (shown below). In this problem you will explore the electric potential associated with this disk at a point P at a location z above the center of the disk.



   (a) (6 points) Draw the appropriate vectors ($\mathbf{r}$, $\mathbf{r}'$, and $\boldsymbol{\mathfrak{r}}$) on the diagram above to indicate how you will set up the electric potential calculation. Explain how you know which vector is which.

   (b) (9 points) Set up the necessary integral to calculate the electric potential at point P. **(You do not need to solve this integral (in fact you can't without more information), but show that you can set it up in a completely integrable form.)**

(c) (6 points) If you calculated this integral for a constant charge density ($\sigma(r, \phi) = \sigma_0$), you would find that the result is:

$$V(z) = \frac{\sigma_0}{2\varepsilon_0}\left[\sqrt{R^2 + z^2} - |z|\right]$$

Show that the units of this equation make sense.

(d) (8 points) What limiting behavior do you expect from this equation as $z \to \infty$? Don't say it goes to zero, how should it go to zero? What is the "small parameter"? Make sure to show your work.

Extra Work (please indicate which part of the quiz you are working)