What is the value of $\int_0^a \sin(n\pi x/a)\sin(m\pi x/a)\ dx$ ?

A. Zero

B. Non-zero

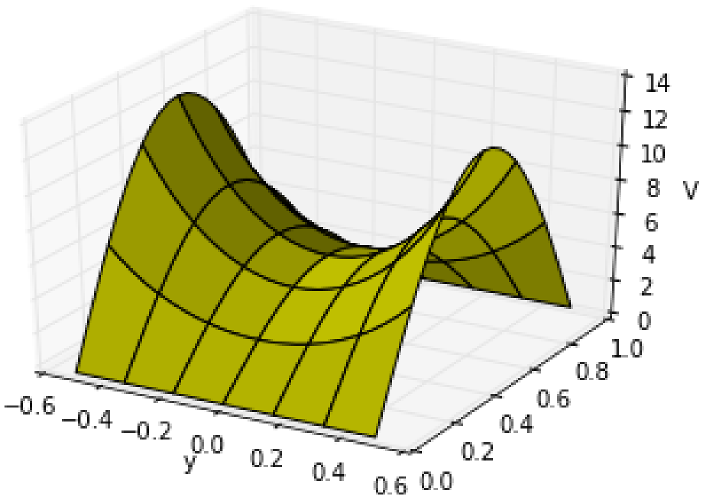
C. Depends on $n$ and $m$

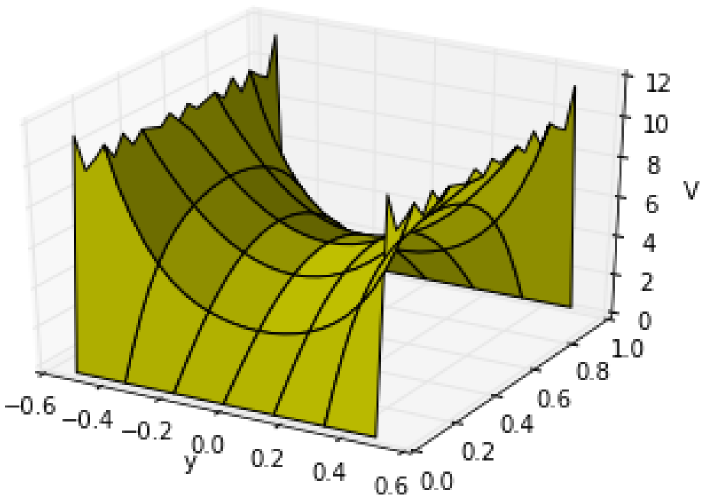
# EXACT SOLUTIONS:

$$V(x,y)=\sum_{n=1}^{\infty}\frac{4V_0}{n\pi}\frac{1}{\cosh\left(\frac{n\pi}{2}\right)}\cosh\left(\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)$$

# APPROXIMATE SOLUTIONS:

# (1 TERM; 20 TERMS)



Given that we want to solve Laplace's equation in 2D and that we have a description for the numerical second derivative of one variable,

$$f''(x) \approx \frac{f(x+a) - 2f(x) + f(x-a)}{a^2}$$

what is the appropriate numerical partial derivative for $V(x, y), \partial^2 V/\partial x^2 \approx$,

A. $[V(x+a) - 2V(x) + V(x-a)]/a^2$

B. $[V(x+a, y) - 2V(x, y) + V(x-a, y)]/a^2$

C. $[V(y+a) - 2V(y) + V(y-a)]/a^2$

D. $[V(x, y+a) - 2V(x, y) + V(x, y-a)]/a^2$

E. More than one is correct

Given that the potential at any point is given by the average of the surrounding points,

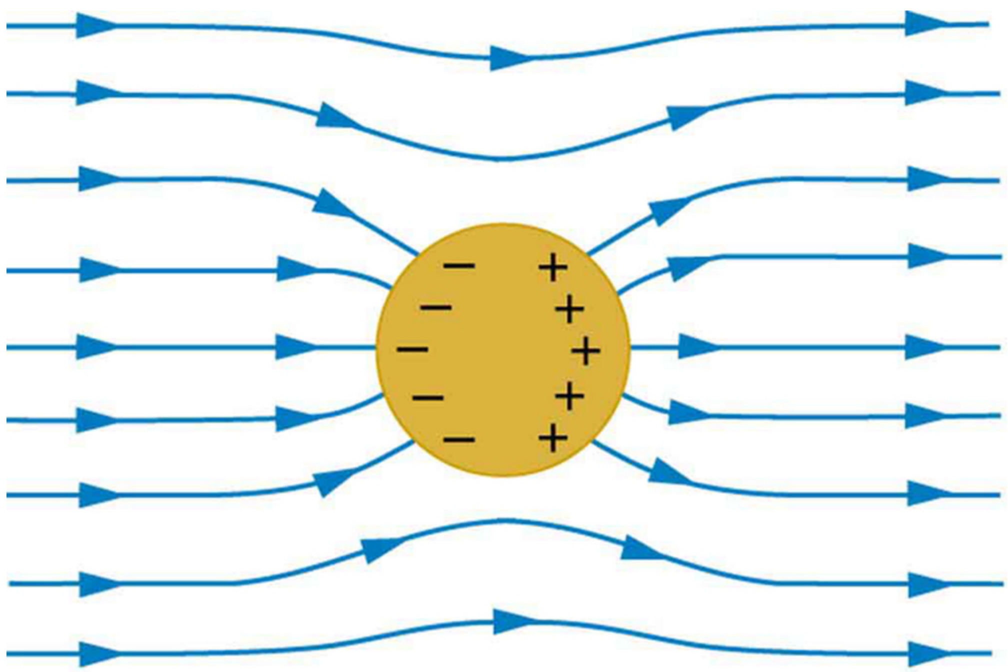
$$V(x, y) \approx \frac{1}{4}[V(x + a, y) + V(x, y + a) \\ +V(x - a, y) + V(x, y - a)]$$

Draft the psuedocode for finding the approximate potential.

Given $\nabla^2 V = 0$ in Cartesian coords, we separated $V(x, y, z) = X(x)Y(y)Z(z)$. Will this approach work in spherical coordinates, i.e. can we separate $V(r, \theta, \phi) = R(r)\Theta(\theta)\Phi(\phi)$?

A. Sure.
B. Not quite - the angular components cannot be isolated, e.g.,$f(r, \theta, \phi) = R(r)Y(\theta, \phi)$
C. It won't work at all because the spherical form of Laplace's Equation has cross terms in it (see the front cover of Griffiths)

# SEPARATION OF VARIABLES (SPHERICAL)



The ODE that describes the $R(r)$ part of our solution is:

$$\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) = l(l+1)R$$

I claim this ODE gives rise to polynomial solutions.

Find a general solution for $R(r)$ in terms of $l$.