energy eigenvalue.

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\varphi_E(x) + \underline{V(x)}\varphi_E(x) = E\varphi_E(x)$$

$$V = \begin{cases} \infty & x<0 \\ 0 & 0<x<L \\ \infty & x>L \end{cases}$$

$E < V_0$ boundstates

$$V = \begin{cases} V_0 & x< -a/2 \\ 0 & -a/2<x<a/2 \\ V_0 & x>a/2 \end{cases}$$

$E < V_0$

$$V = \begin{cases} 0 & x<0 \\ -\beta\delta(x) & x=0 \\ 0 & x>0 \end{cases}$$ δ fun.

Free Particle

$V = 0$ everywhere

$E > V > 0$ unbound states

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\varphi_E(x) = E\varphi_E(x) \quad \text{b/c } V=0$$

eigenvalue

energy eigenvalue eqn

for all x.

$$\frac{d^2\varphi_E(x)}{dx^2} = -\underbrace{\frac{2mE}{\hbar^2}}_{>0}\varphi_E(x)$$

$E>0$

$$k^2 = \frac{2mE}{\hbar^2} > 0$$

$$\boxed{\frac{d^2\varphi_E(x)}{dx^2} = \underbrace{-k^2}_{<0}\varphi_E(x)}$$

1D free particle

oscillator

$$\varphi_E(x) = \underline{A}e^{ikx} + \underline{B}e^{-ikx}$$

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

no such constraints!

any energy is allowed

Solutions, E as parameter.

Bound State inf. sq. well

$k \sim \pi$

δ-fun.

unique sol.

finite sq. well

$-k\cot(ka) = ?$

For a given choice of energy

$$-i\hbar \frac{d}{dt}|\psi(t)\rangle = \hat{H}(t)|\psi(t)\rangle$$

b/c all energies are possible...

$E_0 = \hbar\omega_0$ einstein relationship.

$$\psi(t) = c_n \phi_E(x)\, e^{-iE_0 t/\hbar}$$

$\Leftarrow$ just as we've always done

for choice of E

$$\psi_E(x,t) = \left(Ae^{ikx} + Be^{-ikx}\right)e^{-iE_0t/\hbar}$$

$$= \left(Ae^{ikx} + Be^{-ikx}\right)e^{-i\omega_0 t}$$

$$\psi_E(x,t) = A\underbrace{e^{i(kx-\omega_0 t)}} + B\underbrace{e^{-i(kx+\omega_0 t)}}$$

$e^{\pm i(kx \pm \omega_0 t)} \Rightarrow$ travelling waves

$\lambda = 2\pi/k$

$v = \omega_0/k$

$f(x \pm vt)$

$e^{\pm ik(\underbrace{x \pm \omega_0/k\, t})}$

$E = p^2/2m$

$\lambda = \frac{h}{2mE} \Rightarrow \lambda = \frac{h}{p}$

$$\psi(x,t) = Ae^{i(kx - \omega_0 t)} + Be^{-i(kx + \omega_0 t)}$$

cu

$$\tilde{h}(t) = Ae^{i(kx - \omega_0 t)}$$ mathematical convienecеon

$$h(t) = \mathrm{Re}(\tilde{h})$$ obs

$$\tilde{\vec{E}}(t) = \tilde{\vec{E}}_0 e^{i(kx - \omega t)}$$

$$\vec{E} = \mathrm{Re}(\tilde{\vec{E}})$$ obs

$$P(x) = |\psi(x,t)|^2$$

$$\psi(x,t) = \phi_1(x,t) + \phi_2(x,t) + \phi_3(x,t) \ldots$$

→ imaginary part

→ x variable

→ t variable

$$\psi(x,t) = Ae^{i(kx-\omega_0 t)} + Be^{-i(kx+\omega_0 t)}$$