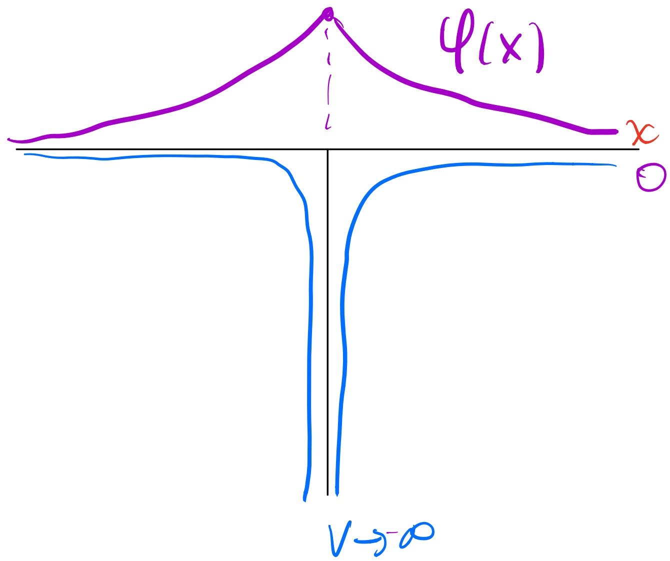

Dirac δ potential

$$V(x) = -\beta\,\delta(x)$$

vanishes loc. of δ functions.

$$\boxed{\psi_E(x) = \begin{cases} A e^{gx} & x<0 \\ A e^{-gx} & x>0 \end{cases} \qquad A = \sqrt{\frac{m\beta}{\hbar}} \qquad E = -m\beta^2/2\hbar^2}$$

only one bound state.

$$\lim_{\varepsilon\to 0}\left(\left.\frac{d\psi}{dx}\right|_{+\varepsilon} - \left.\frac{d\psi}{dx}\right|_{-\varepsilon} = \frac{2m}{\hbar^2}\int_{-\varepsilon}^{\varepsilon} V(x)\,\psi(x)\,dx\right)$$

$\frac{d\psi}{dx}$ continuous except when $V \to \pm\infty$ allow to be $\frac{d\psi}{dx}$ discontinuous

Orthogonality

$$\langle E_i | E_j \rangle = \delta_{ij} \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$$

$$\langle x | \phi_1 \rangle = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$$

$$\langle x | \phi_2 \rangle = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right)$$

$$\int_0^L |\phi_1|^2 dx = 1$$

$$\int_0^L \phi_2^* \phi_1 \, dx = 0$$

continuous description of orthogonality

orthogonal & normal = orthonormal

General Description of Orthog.

$$\int_a^b f_m(x) g_n(x) w(x) \, dx = \delta_{mn} C_n$$

$C_n = 1$ orthonormal functions

$$\frac{1}{C_n}\int_a^b f_m(x)\, g_n(x)\, w(x)\, dx = \delta_{mn}$$

Orthogonality Discussion

$$\int_a^b w(x)\, f_n(x)\, g_m(x)\, dx = \delta_{mn} C_n$$

sin, cos,

$W(x) = 1$ $\quad C_n = \pi$

$[a, b] = 0, 2\pi$

$-\pi, \pi$

$$\boxed{\begin{array}{l} \sin\left(\frac{n\pi x}{a}\right) \quad \cos\left(\frac{n\pi x}{a}\right) \quad W(x) = 1 \\ [-a, a] \qquad C_n = 1 \end{array}}$$

Laguerre Polynomial,

$$\int_0^{\infty} \Gamma(x,1,1] L_n(x) L_m(x)\, dx = \delta_m \binom{n}{n}$$

$$\int_0^{\infty} e^{-x} L_n(x) L_m(x)\, dx = \underline{\delta_{mn}} \Gamma$$

b

a

W(x)

Hydrogen Atom

$$-\frac{\hbar^2}{2m} \nabla^2 \psi(\vec{r}) - \frac{e}{4\pi\varepsilon_0 r} \psi(\vec{r}) = E\psi(\vec{r})$$

3D

Legendre Polynomials

$$\psi(\vec{r}) = R(r)\, \Theta(\theta)\, \Phi(\phi)$$

Laguerre Polynomials

$L_0(x) = 1$

$L_1(x) = -x + 1$

$L_2(x) = \frac{1}{2}(x^2 - 4x + 2)$

$P(\cos\theta)$ $L(r)$ ...