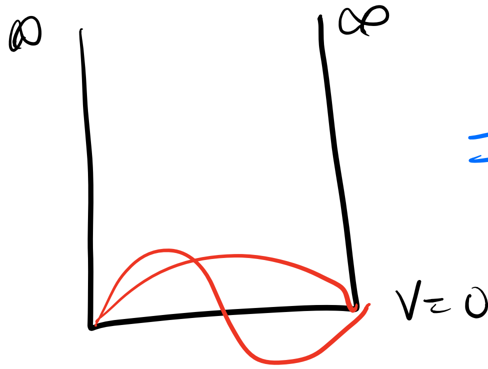
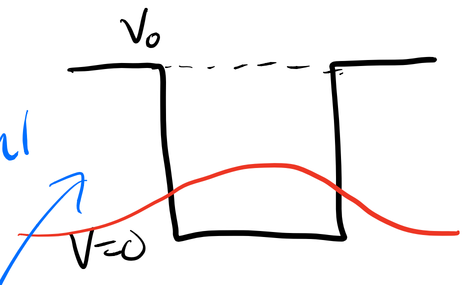
Finite Square Well



⇒ lower potential



classically forbidden

All Bound States

$E < V_0$ Bound States

Ch 5 ⇒ quantized spectrum

$E > V_0$ unbound states

Ch 6 ⇒ continuous energy spectra

$$V = \begin{cases} V_0 & x < -a \quad (1) \\ 0 & -a < x < a \quad (2) \\ V_0 & x > a \quad (1) \end{cases}$$

$$\hat{H}\psi_E(x) = E\psi_E(x)$$

(1) (in the walls)

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\varphi_E(x) + V_0\varphi_E(x) = E\varphi_E(x)$$

(2) (in the well)

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\varphi_E(x) = E\varphi_E(x)$$

interested : $E > 0$ particle positive energy

$E < V_0$ bound states

(1) $$\underline{\frac{d^2}{dx^2}\varphi_E(x) = -\frac{2m(E-V_0)}{\hbar^2}\varphi_E(x)}$$

(2) $$\underbrace{\frac{d^2}{dx^2}\varphi_E(x) = \frac{-2mE}{\hbar^2}\varphi_E(x)}_{\text{Inf Sq. Well}}$$

① $\frac{2m(E-V_0)}{\hbar^2} < 0 \quad E < V_0$

$$q^2 = -\frac{(2m(E-V_0))}{\hbar^2} > 0$$

$$\boxed{\varphi_E''(x) = q^2 \varphi_E(x)}$$

② $\boxed{\varphi_E''(x) = -k^2 \varphi_E(x)}$ $\quad k^2 = \frac{2mE}{\hbar^2}$

$$\varphi_E(x) = \begin{cases} Ae^{qx} + Be^{-qx} & x < -a \\ C\sin(kx) + D\cos(kx) & -a < x < a \\ Fe^{qx} + Ge^{-qx} & x > a \end{cases}$$

3 B.C.s

① $d\varphi_E/dx$ continuous

infinite ΔKE walls
infinite Force

(2) ψ_E continuous

popping in and out existence not cool.

(3) $$\int_{-\infty}^{+\infty} |\psi_E|^2 dx = 1$$

particle has to somewhere

practical prob $|\psi_E|^2$

Prob ≤ 1

$$ka \tan(ka) = qa$$

! non seperable or nonlinear combinations

Solutions?

$\Rightarrow$ root finding (newton's method)

$$f(E) = g(E) - h(E)$$

roots

$\Rightarrow$ taylor expand around root?

$\Rightarrow$ graphing & zooming in

$$z = \sqrt{\frac{2mEa^2}{\hbar^2}} \qquad z_0 = \sqrt{\frac{2mV_0a^2}{\hbar^2}}$$

Strongly bound $\Rightarrow$ $V_0 >> E$

$z_0 >> z$

weakly bound $\Rightarrow$ $E \sim V_0$

$z \sim z_0$

$\sqrt{z_0^2 - z^2} \approx z_0$ find solutions