

Spin  $1/2$  ; Spin 1

+ \_\_\_\_\_ 2 state      + \_\_\_\_\_ 3 states

- \_\_\_\_\_ 2x2 matrices      - \_\_\_\_\_ 3x3 matrices

$\begin{pmatrix} a \\ b \end{pmatrix}$        $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$

① finite states but a lot  
 $\rightarrow 10, 20, 100$   $\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_{100} \end{pmatrix}$   
 100x100

② in finite states

$\hookrightarrow$  position representation state vector  
 $\Rightarrow$  wave function

$$\hat{H} |E_i\rangle = E_i |E_i\rangle$$

↑ Ham operator      ↑ Eigen states      ↑ Eigenval

$$H = T + V \Rightarrow \text{1D} \quad H = \frac{p_x^2}{2m} + V(x)$$

$$\hat{H} = \frac{\hat{p}_x^2}{2m} + V(\hat{x})$$

$$\hat{x} \doteq x \quad \hat{p} = -i\hbar \frac{d}{dx}$$

position rep of operators

$$|\psi\rangle \doteq \psi(x)$$

gen state  
vector

position dep  
cont. function

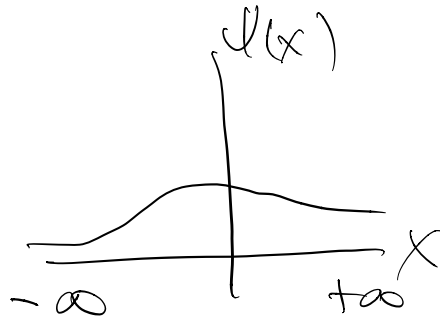
$$|E_i\rangle \doteq \psi_E(x) \quad \text{energy eigenstate}$$

Energy Eigenvalue Problem

$$\left[ \frac{-\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) \right] \psi_E(x) = E \psi_E(x)$$

## Probability function

$$P(x) = |\psi(x)|^2$$



$$1 = \int_{-\infty}^{\infty} P(x) dx = \int_{-\infty}^{+\infty} |\psi(x)|^2 dx$$

normalization

## Probability Energy States

$$P_{E_n} = |\langle E_n | \psi \rangle|^2 = \left| \int_{-\infty}^{\infty} \phi_n^*(x) \psi(x) dx \right|^2$$

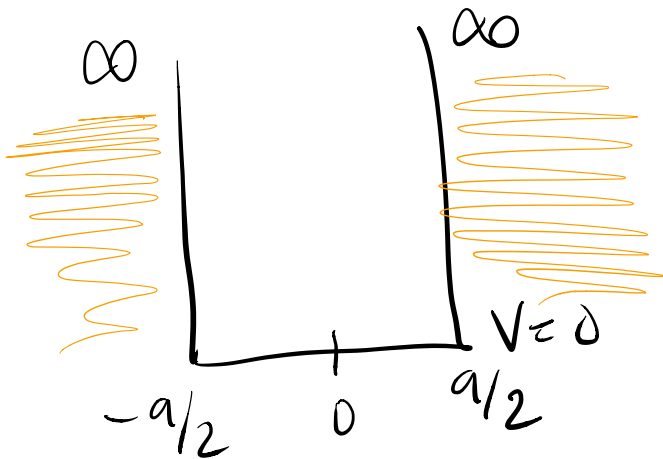
## Expectation Value

$$\langle x \rangle = \langle \psi | x | \psi \rangle = \int_{-\infty}^{\infty} \psi^* x \psi dx$$

# Infinite Square Well

$$V = \begin{cases} \infty & x < -a/2 \\ 0 & -a/2 < x < a/2 \\ \infty & x > a/2 \end{cases}$$

$$-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \psi_E(x) + V(x) \psi_E(x) = E \psi_E(x)$$



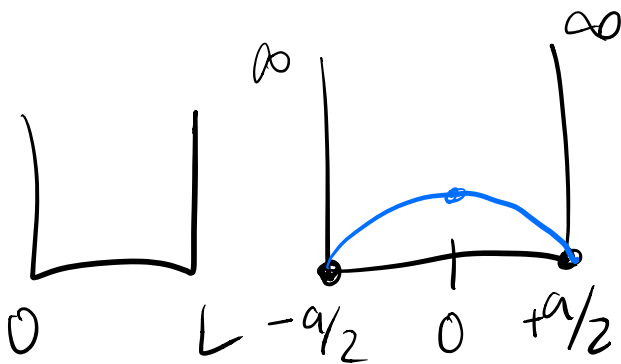
$$-\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} = E \psi$$

$$k^2 \equiv \frac{2mE}{\hbar^2}$$

$$\frac{d^2}{dx^2} \psi(x) = -k^2 \psi(x)$$

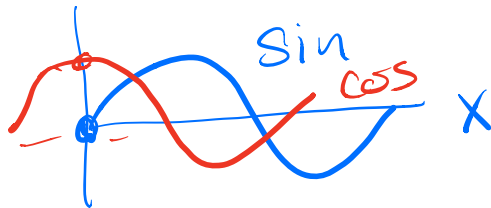
$$\psi(x) = A e^{-ikx} + B e^{+ikx} \quad -k^2 > 0$$

$$\psi(x) = C \cos(kx) + D \sin(kx)$$



$$\psi(x = \pm a/2) = 0$$

Symmetric  
around zero



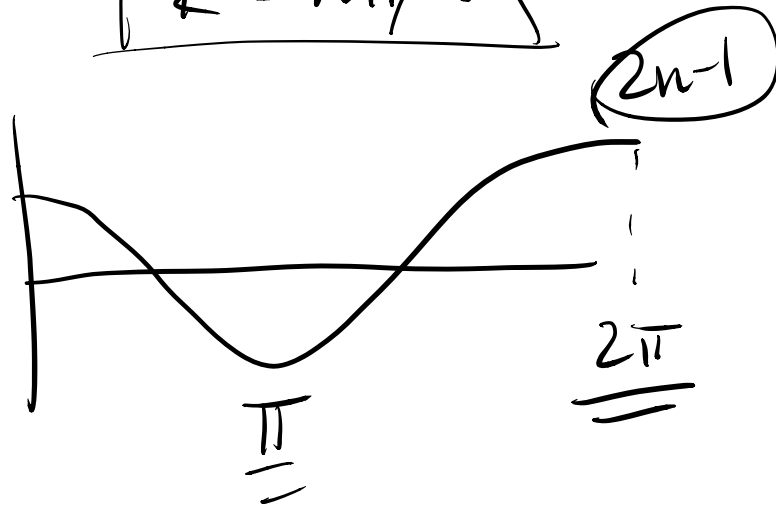
$$\psi = C \cos(kx)$$

$$\cos\left(k \frac{a}{2}\right) = 0$$

$$\cos\left(\frac{n\pi}{2}\right) = 0$$

$$k = n\pi/a$$

$$n = 1, 3, 5, \dots$$



$$k_n = \sqrt{\frac{2mE_n}{\hbar^2}}$$

$$k_n = \frac{n\pi}{a}$$

$$n = 1, 3, 5, \dots$$

$$\hookrightarrow \psi_n(x) = C \cos(k_n x)$$