Example: Spin 1 system

$\vec{B}_0 = B_0 \hat{z}$ & $\vec{B}_2 = B_2 \hat{x}$ $\qquad B_2 \ll B_0$

$$H_0 \doteq \begin{pmatrix} \hbar\omega_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\hbar\omega_0 \end{pmatrix} \qquad \begin{matrix} E_+^{(0)} = \hbar\omega_0 & |+\rangle \\ E_0^{(0)} = 0 & |0\rangle \\ E_-^{(0)} = -\hbar\omega_0 & |-\rangle \end{matrix}$$

$$H' = \omega_2 S_x \doteq \frac{\hbar\omega_2}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$E_n^{(1)} = 0$ b/c $H'_{nn} = 0$

$\langle n^{(0)} | H' | n^{(0)} \rangle = E_n^{(1)}$

$\langle -1|, \langle 0|$ $\qquad |+\rangle$

$$E_n^{(2)} = \sum_{m \neq n} \frac{|\langle n^{(0)} | H' | m^{(0)} \rangle|^2}{(E_n^{(0)} - E_m^{(0)})}$$

$$E_+^{(2)} = \left.\frac{|\langle -^{(0)} | H' | +^{(0)} \rangle|^2}{(E_-^{(0)} - E_+^{(0)})}\right\} 0$$

$$+ |\langle 0^{(0)} | H' | +^{(0)} \rangle|^2 / (E_0^{(0)} - E_+^{(0)})$$

$$E_-^{(0)} - E_+^{(0)} = -2\hbar\omega_0$$

$$E_0^{(0)} - E_+^{(0)} = -\hbar\omega_0$$

$$\langle - | H' | + \rangle = 0$$

$$\langle 0 | H' | + \rangle = \hbar\omega_2/\sqrt{2}$$

$$H' = \frac{\hbar\omega_2}{\sqrt{2}} \begin{array}{c} \begin{array}{ccc} |+\rangle & |0\rangle & |-\rangle \end{array} \\ \begin{pmatrix} \cancel{0} & 1 & 0 \\ 1 & \cancel{0} & 1 \\ 0 & 1 & \cancel{0} \end{pmatrix} \end{array} \begin{array}{c} |+\rangle \\ |0\rangle \\ |-\rangle \end{array}$$

$$H_0 = \begin{array}{c} \\ |+\rangle \\ |0\rangle \\ |-\rangle \end{array} \begin{array}{c} \begin{array}{ccc} |+\rangle & |0\rangle & |-\rangle \end{array} \\ \begin{pmatrix} h\omega_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\hbar\omega_0 \end{pmatrix} \end{array}$$

$$E_+^{(2)} = \frac{|\langle 0 | H' | + \rangle|^2}{(E_+^{(0)} - E_0^{(0)})} = \frac{(\hbar\omega_2/\sqrt{2})^2}{\hbar\omega_0}$$

$$\boxed{E_+^{(2)} = \frac{\hbar}{\sqrt{2}} \frac{\omega_2^2}{\omega_0}}$$

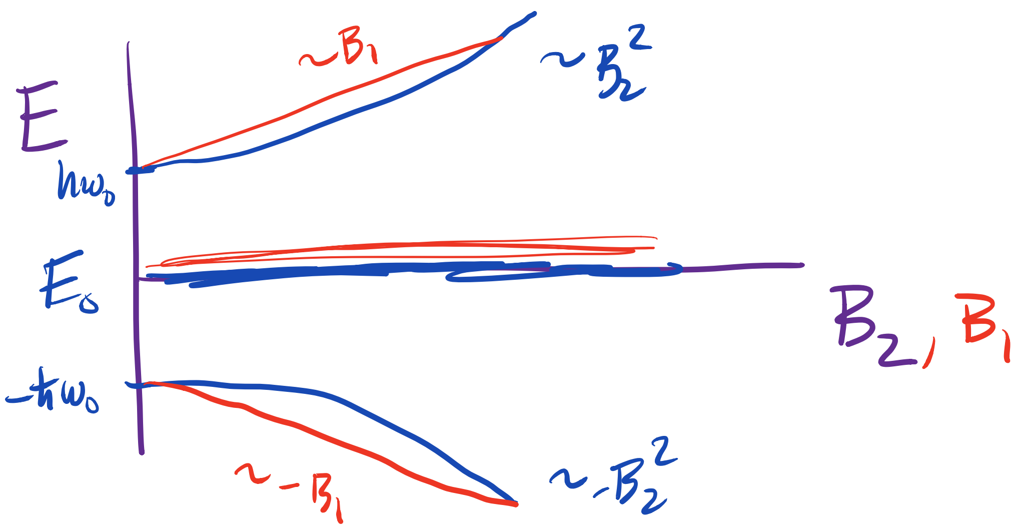
$$E_n^{(2)} = \sum_{m \neq n} \frac{|\langle n^{(0)} | H' | m^{(0)} \rangle|^2}{\left(E_n^{(0)} - E_m^{(0)}\right)}$$

$$E_+ \simeq \hbar\omega_0 + \frac{\hbar}{\sqrt{2}} \frac{\omega_2^2}{\omega_0}$$

$$E_0 \simeq 0 + 0 + 0 = 0$$

$$E_- \simeq -\hbar\omega_0 - \frac{\hbar}{\sqrt{2}} \frac{\omega_2^2}{\omega_0}$$

$\omega_2 \sim B_2$



$$\begin{cases} \vec{B}_1 = B_1 \hat{z} \Rightarrow \text{linear energy corr.} \\ B_2 = B_2 \hat{x} \Rightarrow \text{quadratic energy corr.} \end{cases}$$

$$\omega_1 S_z = \underline{H'} \doteq \begin{pmatrix} \hbar\omega_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\hbar\omega_1 \end{pmatrix}$$

$$\omega_2 S_x = H' \doteq \begin{pmatrix} 0 & \hbar\omega_2/\sqrt{2} & 0 \\ \hbar\omega_2/\sqrt{2} & 0 & \hbar\omega_2/\sqrt{2} \\ 0 & \hbar\omega_2/\sqrt{2} & 0 \end{pmatrix}$$

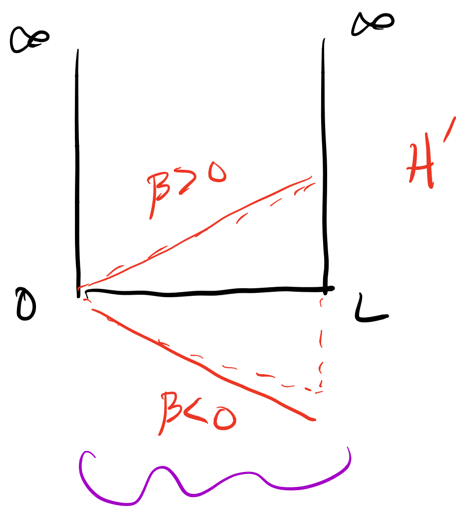
Perturb Infinite Square Well

$$\underline{H_0 = \frac{-\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)}$$

$$V(x) = \begin{cases} 0 & 0 < x < L \\ \infty & x > L \\ & x < 0 \end{cases}$$

$$E_n^{(0)} = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

$$|n^{(0)}\rangle \doteq \varphi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$$



$$H = \frac{-\hbar^2}{2m}\frac{d^2}{dx^2} + \beta x$$

$$V(x) = \begin{cases} \infty & x<0 \\ & x>L \\ \beta x & 0<x<L \end{cases}$$

$$H = H_0 + H' \Rightarrow H' = \begin{cases} \beta x & \underline{0<x<L} \\ 0 & \underline{\text{otherwise}} \end{cases}$$