Position Rep of the QHO

$$H|n\rangle = (n+\tfrac{1}{2})\hbar\omega|n\rangle$$

$$\langle m|n\rangle = \delta_{m,n}$$

$$\hat{H} = \hbar\omega(a^\dagger a + \tfrac{1}{2})$$

$$\hbar\omega(a^\dagger a)|n\rangle + \underbrace{\tfrac{1}{2}\hbar\omega|n\rangle}$$

$$= (n+\tfrac{1}{2})\hbar\omega|n\rangle$$

$$= n\hbar\omega|n\rangle + \underbrace{\tfrac{1}{2}\hbar\omega|n\rangle}$$

$$\boxed{a^\dagger a|n\rangle = n|n\rangle}$$

$N \equiv a^\dagger a$ number operator

$\langle x|0\rangle \doteq \varphi_0(x)$

$$\frac{d\varphi_0(x)}{dx} = \frac{-m\omega}{\hbar}\varphi_0(x)$$

$$\varphi_0(x) = A e^{-\alpha x^2}$$

✓

$$\alpha = \frac{m\omega}{2\hbar}$$

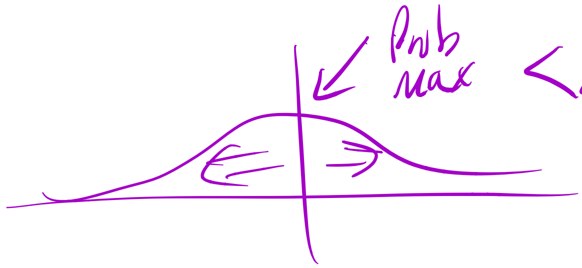
$$\varphi_0(x) = A e^{-m\omega x^2/2\hbar}$$

$$1 = \int_{-\infty}^{\infty} |\varphi_0(x)|^2 dx \Rightarrow |A|^2 = \sqrt{\frac{m\omega}{\pi\hbar}}$$

Choose real & positive A

$$\boxed{\varphi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega x^2/2\hbar}}$$

$\doteq \langle x|0\rangle$



$\langle x \rangle = 0$

$$\big|a|n\rangle\big|^2 = (\langle n|a^\dagger)(a|n\rangle)$$

$$= \langle n|a^\dagger a|n\rangle = \langle n|N|n\rangle = n\langle n|n\rangle$$

$$= \langle n|a^\dagger c|n-1\rangle = c\langle n|a^\dagger|n-1\rangle$$

$$= c\langle n|c|n\rangle = |c|^2\langle n|n\rangle$$

$$\big|a|n\rangle\big|^2 = \big|c|n-1\rangle\big|^2$$

$$\big|a|n\rangle\big|^2 = n \qquad |c|^2 = n$$

$$\boxed{a|n\rangle = \sqrt{n}\,|n-1\rangle}$$

$$\left|a^\dagger|n\rangle\right|^2 = \left(\langle n|a\right)\left(a^\dagger|n\rangle\right)$$

$$[a,a^\dagger] = aa^\dagger - a^\dagger a = 1$$

$$\boxed{aa^\dagger = 1 + a^\dagger a} = 1 + N$$

$$\langle n|a\,a^\dagger|n\rangle = \langle n|1|n\rangle + \langle n|N|n\rangle$$

$$= n+1$$

$$a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

$$a|n\rangle = \sqrt{n}\,|n-1\rangle$$

$$|n-1\rangle = \frac{a}{\sqrt{n}}|n\rangle$$ normalized ket $|n-1\rangle$

$$a^{+}|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

$$|n+1\rangle = \frac{a^{+}}{\sqrt{n+1}}|n\rangle$$ normalized ket $|n+1\rangle$

$$\varphi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega x^2/2\hbar}$$

$$\varphi_1(x) = \frac{a^{+}}{\sqrt{1}}\varphi_0(x)$$

$$\varphi_2(x) = \frac{a^{+}}{\sqrt{2}}\varphi_1(x) = \frac{a^{+}}{\sqrt{2}}\frac{a^{+}}{\sqrt{1}}\varphi_0(x)$$

$$\psi_2(x) = \frac{(a^+)^2}{\sqrt{2\cdot 1}} \psi_0(x)$$

$$\boxed{\psi_n(x) = \frac{(a^+)^n}{\sqrt{n!}} \psi_0(x)}$$

$$a^+ \equiv \sqrt{\frac{m\omega}{2\hbar}} \left(x - \frac{\hbar}{m\omega} \frac{d}{dx} \right)$$

$$\boxed{\psi_n(x) = \frac{1}{\sqrt{n!}} \left(\sqrt{\frac{m\omega}{2\hbar}} \right)^n \left(x - \frac{\hbar}{m\omega} \frac{d}{dx} \right)^n \psi_0(x)}$$

$$\boxed{|n\rangle = \frac{1}{\sqrt{n!}} (a^+)^n |0\rangle}$$