Quantum Harmonic Oscillator

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2$$

$$\hat{H}|E\rangle = E|E\rangle$$

$\hat{H} \rightarrow$ a lowering

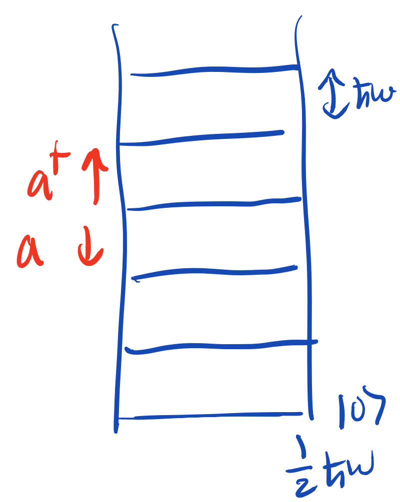
$a^\dagger$ raising

$$a \equiv \sqrt{\frac{m\omega}{2\hbar}} \left(\hat{x} + i \frac{\hat{p}}{m\omega}\right)$$

$$a^\dagger \equiv \sqrt{\frac{m\omega}{2\hbar}} \left(\hat{x} - i \frac{\hat{p}}{m\omega}\right)$$

$$\hat{H} = \hbar\omega \left(a^\dagger a + \frac{1}{2}\right)$$

$$\hat{H}|n\rangle = \left(n + \frac{1}{2}\right)\hbar\omega |n\rangle$$



$$E_{ground} = \frac{1}{2}\hbar\omega$$

$$E_n = \left(n+\frac{1}{2}\right)\hbar\omega \qquad n = 0, 1, 2, \ldots \rightarrow$$

$$H|n\rangle = \left(n+\frac{1}{2}\right)\hbar\omega|n\rangle$$

$$\langle n|n\rangle = 1 \qquad \langle m|n\rangle = \delta_{m,n}$$

$$\langle n|a^{\dagger}a|n\rangle = \langle n|\underline{N}|n\rangle$$

$$= \langle n|n|n\rangle = n\langle n|n\rangle = n$$

$$\underline{a|0\rangle = 0} \qquad \langle x|n\rangle \doteq \varphi_n(x)$$

$$\varphi_0(x)$$

$$\rightarrow a\varphi_0(x) = 0$$

$$\rightarrow \left(x + c\frac{d}{dx}\right)\varphi_0(x) = 0$$

$$\boxed{\frac{d\psi_0(x)}{dx} = -\frac{m\omega}{\hbar}\, x\, \psi_0(x)}$$

~~not~~ $e^{\pm x^2}$

$$\psi_0(x) = Ae^{-\alpha x^2}$$

$Ae^{+\alpha x^2}$?

normalization problem

$$\alpha = \frac{m\omega}{2\hbar} \quad ?$$

$$\psi_0(x) = Ae^{-m\omega x^2/2\hbar}$$

$$|A| = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}$$

$$\boxed{\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega x^2/2\hbar}}$$

prob

$\langle x\rangle = 0$