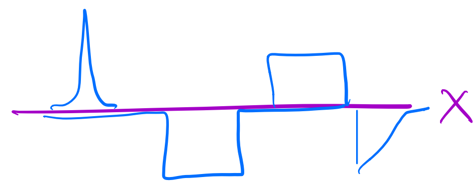
1D Wells



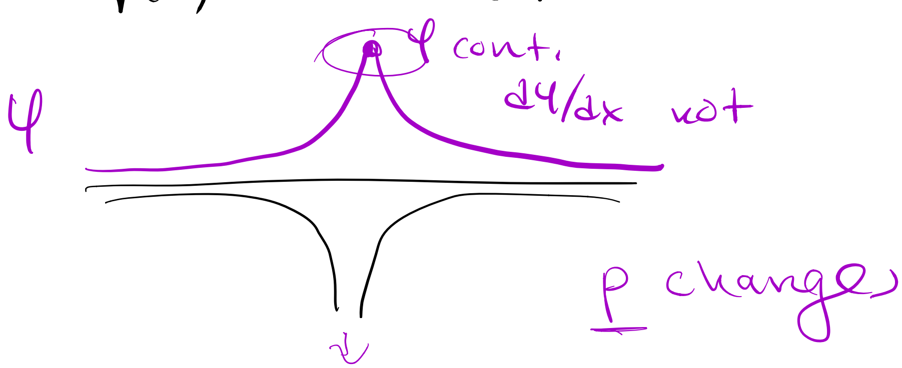
1. Sketch the well
2. Write $H|E\rangle = E|E\rangle$ for each region
3. Determine general sol'ns. (know E, V_0 relations)
4. Match BC's. ← φ continuous → $\frac{d\varphi}{dx}\Big|$ continuous unless $V \to \infty$
5. Find E or E relationship

⟹ alot of algebra

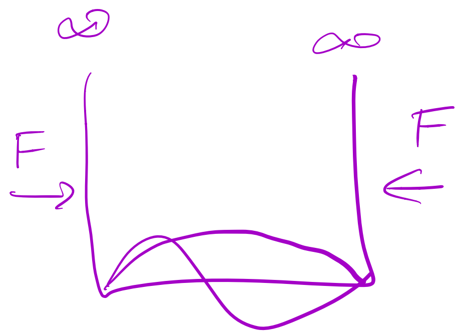
$$\lim_{\varepsilon \to 0} \left\{ \frac{d\varphi_E}{dx}\bigg|_{L+\varepsilon} - \frac{d\varphi_E}{dx}\bigg|_{L-\varepsilon} = +\frac{\hbar^2}{2m}\int_{L+\varepsilon}^{L-\varepsilon} V(x)\varphi_E(x)\,dx \right\}$$

$V(x)$ well behaved → goes to zero.

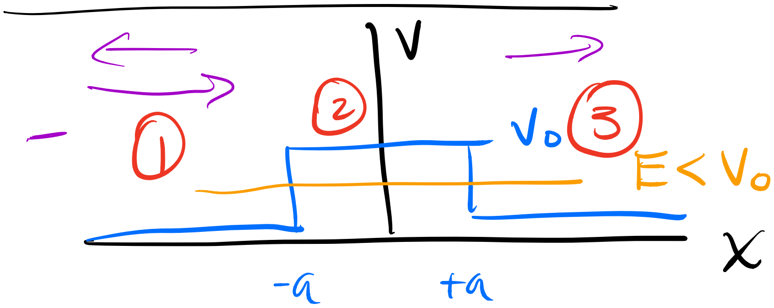
$V(x) \to \infty \quad L+\varepsilon \to L-\varepsilon$



$\hat{p} \propto \frac{\partial}{\partial x}$ $\qquad$ $\frac{dV}{dx} \propto F$



Ex: Potential Barrier



1 & 3

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi_E = E\psi_E \qquad \frac{d^2}{dx^2}\psi_E = -\frac{2mE}{\hbar^2}\psi_E$$

$$k^2 = \frac{2mE}{\hbar^2} > 0$$

2

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_0\right)\psi_E = E\psi_E$$

$$\frac{d^2}{dx^2}\varphi_E = \overset{(-)}{\frac{-2m}{\hbar^2}}\,\underbrace{\overset{(-)}{(E-V_0)}}_{E<V_0}\,\varphi_E$$

$$\frac{d^2\varphi_E}{dx^2} = \underbrace{\frac{2m}{\hbar^2}(V_0-E)}_{q^2>0}\,\varphi_E$$

$$\varphi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx} & x<-a \\ \boxed{Ce^{qx} + De^{-qx}} & -a<x<a \\ Fe^{ikx} + \cancel{Ge^{-ikx}} & x>a \end{cases}$$

$G=0$ no particles to left

$$T = \frac{|F|^2}{|A|^2} \qquad R = \frac{|B|^2}{|A|^2}$$

$\leftarrow$ eliminate C & D for A, B, F

$\varphi(-a)$ & $\varphi(a)$ are continuous

$$\left.\frac{d\psi}{dx}\right|_{\pm a} \text{ is continuous}$$

$$T = \frac{1}{1 + \frac{(k^2+q^2)^2 \sinh^2(2qa)}{4k^2q^2}}$$

$$R = 1 - T = \frac{1}{1 + \frac{4k^2q^2}{(k^2+q^2)\sinh^2(2qa)}}$$

high E

$T \to 1$ $\quad$ $R \to 0$

set/ $2qa = n\pi$ $\to$ energy resonances

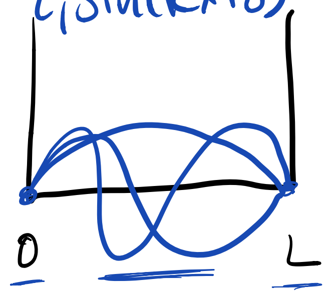
Delta Functions $\to$ BCs.

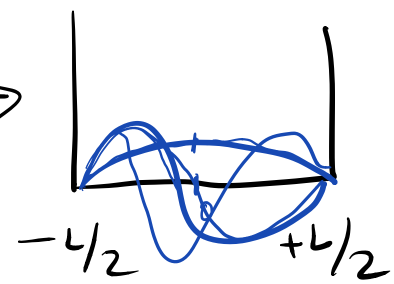
Match BC's in general

Delta func $\to$ Normalization

⇒ Energy eigenstates vs. position rep of eigenstates

$c_1 \sin(kx+\delta)$

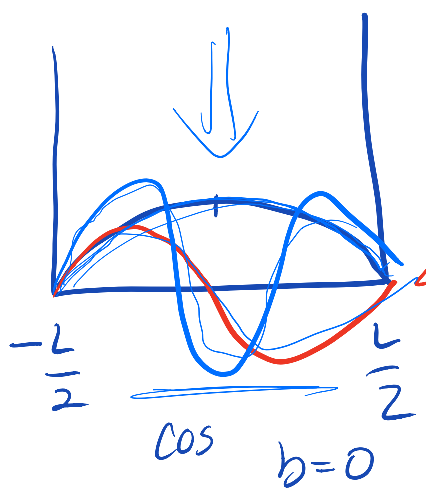




$$\varphi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx} \\ a\cos(kx) + b\sin(kx) \Leftarrow \\ c_1 \sin(kx+\delta) \Leftarrow \\ c_2 \cos(kx+\delta) \end{cases}$$

vs?

$$a\cos(kx) + b\sin(kx) = c_1 \sin(kx+\delta)$$



$a\cos(kx)$

all a_n's different

only two diff δ's → even
→ odd

ket notation. abstract notation

$|E_n\rangle \Leftarrow$ energy eigenstate for some QM sys.

$$\boxed{H|E_n\rangle = E_n|E_n\rangle}$$

specific inf. square well

$$\varphi_n(x) = \langle x|E_n\rangle = \boxed{\sqrt{\frac{2}{2}}\sin\left(\frac{n\pi x}{2}\right)}$$

project onto x basis

$$H_{sys}\varphi_n(x) = -\frac{\hbar}{2m}\frac{d^2}{dx^2}\varphi_n(x)$$

$$= E_n\varphi_n(x)$$

$$\phi_p(p) = \langle p|E_n\rangle$$

project onto p basis

if you can: work w/ **kets!**

$$\boxed{\begin{aligned} |\psi\rangle &= a|E_1\rangle + b|E_2\rangle + c \ldots \\ |\psi\rangle &= a|p_1\rangle + b|p_2\rangle \ldots \end{aligned}}$$

$\hat{p}$ $\quad$ $\hat{H}$ $\quad$ $\hat{x}$