Position & Momentum Representations

$\psi(x)$ position rep → ("wave function")

$\phi(p)$ momentum rep → ("momentum distribution")

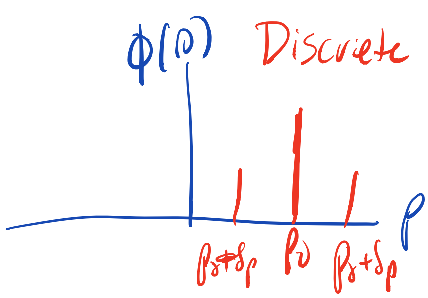
↳ ("momentum space wave function")

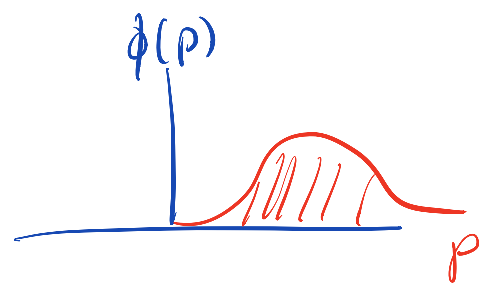
$$\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{+\infty} \phi(p)\, e^{ipx}\, dx$$

$$\phi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{+\infty} \psi(x)\, e^{-ipx}\, dp$$

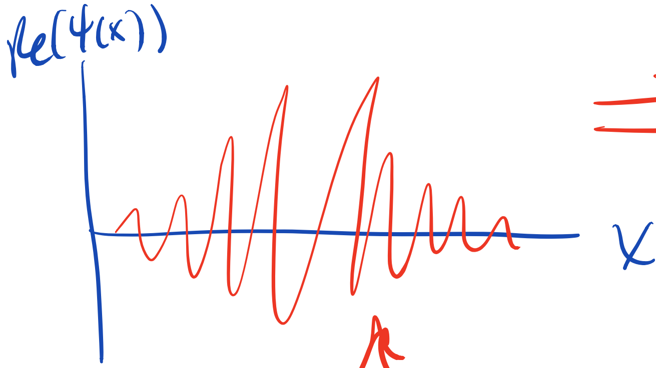
Fourier Transform

have $\phi(p)$ ⇒ know p is distribute





construct $\psi(x)$ using F.T.

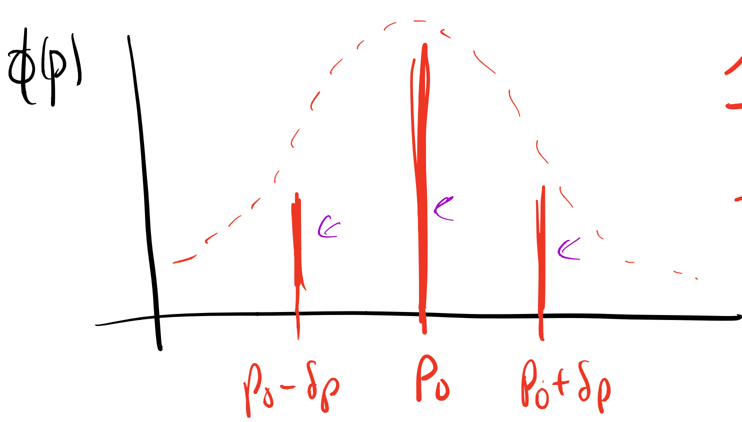


$\Rightarrow$ inverse F.T.

find $\phi(p)$
give rise
to $\psi(x)$

$\psi(x)$	$\phi(p)$	$\varphi(x)$
$\Uparrow$	$\Uparrow$	$\Uparrow$
wave fun. gen state	mom. space wave.	eigen Energy states (position rep)

<u>Example: 3 momentum wave</u>



$1 \, p_0$

$\frac{1}{2}(p_0 \pm \delta p)$

$$\psi(x,0) = \sum_j c_j \phi_{p_j}(x)$$

Free particle
mom. e-states
are E. e-states

$$= \sum_j c_j \frac{1}{\sqrt{2\pi\hbar}} e^{ip_jx/\hbar}$$

1 p_0 1/2 $p_0 \pm \delta p$

$$\boxed{= \frac{1}{\sqrt{2\pi\hbar}} \left[\frac{1}{2} e^{\underline{i(p_0-\delta p)x/\hbar}} + e^{\underline{ip_0x/\hbar}} + \frac{1}{2} e^{\underline{i(p_0+\delta p)x/\hbar}} \right]}$$

$$\times e^{-iE_jt/\hbar}$$

common method
for time dep.

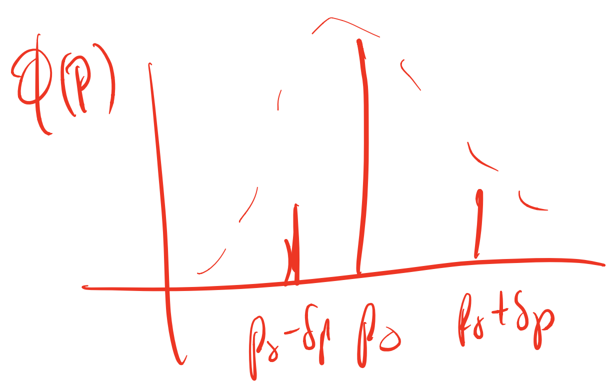
$$\boxed{E_j = \frac{p_j^2}{2m}}$$

$$E_{p_0} = \frac{p_0^2}{2m} \qquad E_{p_0 \pm \delta p} = \frac{(p_0 \pm \delta p)^2}{2m}$$

$\delta p \ll p_0 \qquad (p_0 \pm \delta p)^2 \simeq p_0^2 \pm 2 p_0 \delta p$

lineritation $\Longleftarrow$ toss out $(\delta p)^2$ terms

$$\boxed{\psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}}\left[\frac{1}{2}\underline{e^{i(p_0-\delta p)x/\hbar}\, e^{-i(p_0^2-2p_0\delta p)t/2m\hbar}} + \underline{e^{ip_0 x/\hbar}\, e^{-ip_0^2 t/2m\hbar}} + \frac{1}{2}\underline{e^{i(p_0+\delta p)x/\hbar}\, e^{-i(p_0^2+2p_0\delta p)t/2m\hbar}}\right]}$$



$$\psi(x,t) = \left(\frac{1}{\sqrt{2\pi\hbar}}\, \underbrace{e^{ip_0x}\, e^{-ip_0^2 t/2m\hbar}}_{\text{wave 1}}\right)$$

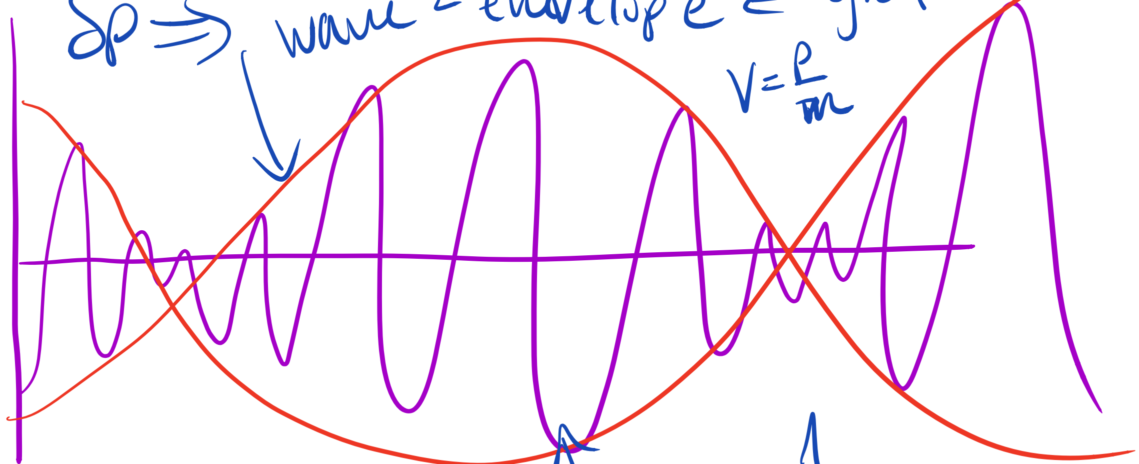
$$\cdot \left(1+\cos\left(\underbrace{\frac{\delta p}{\hbar}x - \frac{p_0\,\delta p}{m\hbar}t}_{\text{wave 2}}\right)\right) \qquad (x-vt)$$

wave 1: carrier wave

wave 2: envelope wave

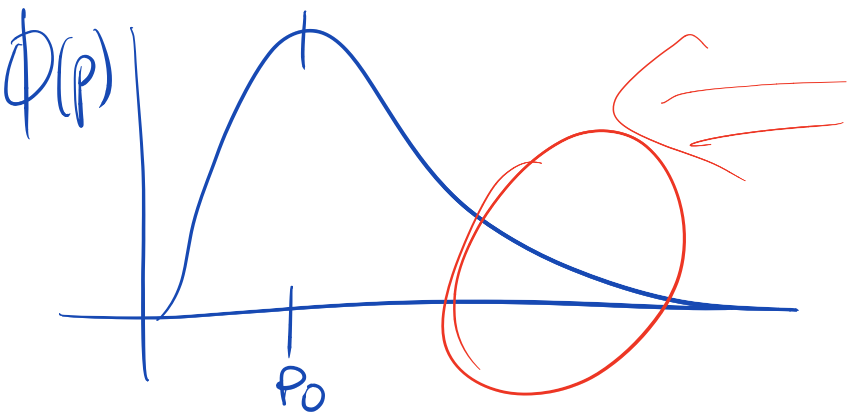
$\delta p \Rightarrow$ wave 2 envelope $\Leftarrow$ group velocity

$v = \frac{p}{m}$

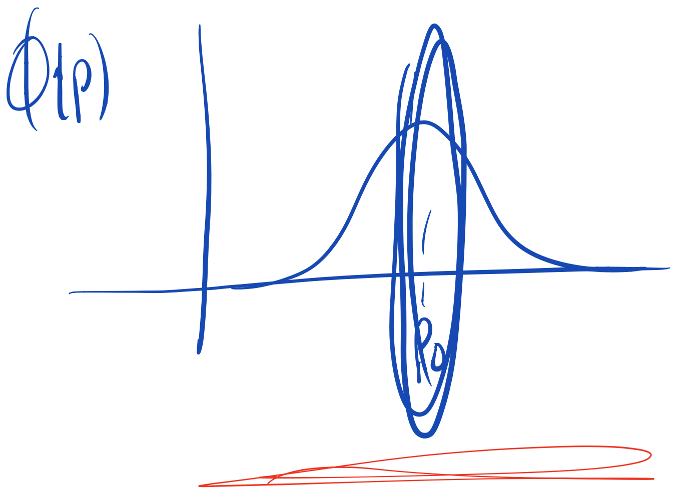


wave 1

$p_0 \Rightarrow$ carrier (phase velocity $v = p/2m$)



$\phi(p)$
p_0



$\phi(p)$
p_0