McIntyre shows how to perform a approximate calculation for the energy eigenstates of a perturbed spin 1/2 system. Rather that rehash this derivation, we will apply our general results to show the same results.

$\boxed{}$

Two level Results, (keep λ for order, set $\lambda=1$ later)

$$E_1 \simeq E_1^{(0)} + \lambda H_{11}' + \lambda^2 \frac{|H_{12}'|^2}{(E_1^{(0)} - E_2^{(0)})}$$

$$E_2 \simeq E_2^{(0)} + \lambda H_{22}' + \lambda^2 \frac{|H_{21}'|^2}{(E_2^{(0)} - E_1^{(0)})}$$

Assume: a spin 1/2 particle in a field $\vec{B}_{tot}$

$$\vec{B}_{tot} = \underbrace{B_0 \hat{z}} + \underbrace{B_1 \hat{z} + B_2 \hat{x}} \quad \text{with}$$

$B_0 >> B_1$ & B_2

$$H_0 = \underline{w_0 S_z} \quad \text{original}$$

$$H' = \underline{w_1 S_z + w_2 S_x} \quad \text{perturbation}$$

2

$$H_0 = -\vec{\mu}\cdot\vec{B}_0 = \omega_0 S_z \doteq \frac{\hbar}{2}\begin{pmatrix} \omega_0 & 0 \\ 0 & -\omega_0 \end{pmatrix}$$

$$H = -\mu\cdot\vec{B}_{tot} = \omega_0 S_z + \omega_1 S_z + \omega_2 S_x$$

$$H \doteq \frac{\hbar}{2}\begin{pmatrix} \omega_0+\omega_1 & \omega_2 \\ \omega_2 & -\omega_0-\omega_1 \end{pmatrix}$$

We can write this as,

$$H = H_0 + H' = \frac{\hbar}{2}\begin{pmatrix} \omega_0 & 0 \\ 0 & -\omega_0 \end{pmatrix} + \frac{\hbar}{2}\begin{pmatrix} \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 \end{pmatrix}$$

Now we use the prescription,

$$E_+ \simeq E_+^{(0)} + \lambda H'_{11} + \lambda^2 \frac{|H_{12}|^2}{(E_+^{(0)} - E_-^{(0)})}$$

$$= \frac{\hbar}{2}\omega_0 + \lambda\frac{\hbar}{2}\omega_1 + \lambda^2 \frac{\hbar^2\omega_2^2/4}{\left(\frac{\hbar}{2}\omega_0 - -\frac{\hbar}{2}\omega_0\right)}$$

just read matrix elements

$$= \frac{\hbar}{2}\omega_0 + \lambda\frac{\hbar}{2}\omega_1 + \lambda^2 \frac{\hbar^2\omega_2^2/4}{\hbar\omega_0}$$

$$E_+ \simeq \frac{\hbar}{2}\omega_0 + \lambda\frac{\hbar}{2}\omega_1 + \lambda^2 \frac{\hbar\omega_2^2}{4\omega_0}$$

Set $\lambda = 1$,

$$\boxed{E_+ \simeq \frac{\hbar}{2}\left(\omega_0 + \omega_1 + \frac{1}{2}\omega_2^2/\omega_0\right) \quad \text{same as McIntyre}}$$

We can do the same for E_-, ③

$$E_- \simeq E_-^{(0)} + \lambda H_{22}' + \lambda^2 \frac{|H_{21}'|^2}{\left(E_2^{(0)} - E_1^{(0)}\right)}$$

$$= -\frac{\hbar}{2}\omega_0 + \lambda \frac{\hbar}{2}(-\omega_1) + \lambda^2 \frac{\hbar^2\omega_2^2/4}{\left(-\frac{\hbar\omega_0}{2} - \frac{\hbar\omega_0}{2}\right)}$$

$$E_- \simeq -\frac{\hbar}{2}\omega_0 - \lambda\frac{\hbar}{2}\omega_1 - \lambda^2 \frac{\hbar\omega_2^2}{4\omega_0}$$

Set $\lambda = 1$,

$$\boxed{E_- \simeq -\frac{\hbar}{2}\left(\omega_0 + \omega_1 + \frac{1}{2}\omega_2^2/\omega_0\right) \quad \text{same as McIntyre}}$$

McIntyre goes into how to obtain approx new eigenstates. But we will reserve that to the formal analysis.