Homework 13 (Due Apr 21st)

Homework 13 focuses on time independent perturbation theory and explores two examples we have explored in detail: The Infinite Square Well and the Quantum Harmonic Oscillator.

For this homework, you are welcome to turn it in by midnight on Sunday Apr. 25th

[PDF]

1. Perturbing the Infinite Square Well

An inifinite square well has walls at $x = 0$ and $x = L$. The potential is zero in the well and rises to infinity at the walls.

We perturb the well with a delta function potential in the middle of the well. That is, $H' = LV_0\delta(x - L/2)$.

1. Determine the first-order correction to the energy for the nth state of the well.
2. You likely found that the energy correction is different for even and odd n. What physical reason would there be for why those corrections would be different?
3. Find the first-order correction to the ground state wavefunction. Which state provides the largest correction?

   Let's remove the delta function perturbation and instead replace with with a rectangular barrier that is centered on the well and is of length ϵL where $0 < \epsilon < 1$ and of height V_0/ϵ.
4. Sketch the setup.
5. Calculate the first-order correction to the energy of the ground state.
6. Compare your answers in part 5 and part 1 in the limit that $\epsilon \to 0$. Discuss the result.

2. Perturbing the QHO

Consider a particle bound in a harmonic potential with $V = \frac{1}{2}m\omega x^2$. We perturb the harmonic well with an anharmonic potential: $H' = \gamma x^3$.

1. Determine the first order corrections to the energy. This should require a direct calculation, use a symmetry argument instead.
2. Consider the second order correction. Here we have to use the off-diagonal matrix elements of the perturbation Hamiltonian. That involves a x^3. Using the ket formulation is likely the easiest approach, so write down H' in terms of the ladder operators.
3. Calculate the second-order energy corrections to first three states.
4. Find the first order corrections to the first three eigenstates.