Example:

consider $\dot{x} = -x + x^3$

$\dot{y} = -2y$

Let's classify the fixed pts.

$\dot{x} = 0 = -x + x^3 = (x^2-1)x$

$0 = (x^2-1)x = (x+1)(x-1)x$

$x \Rightarrow \begin{matrix} +1 \\ -1 \\ 0 \end{matrix}$

$\dot{y} = 0 = -2y \qquad y = 0$

Three fixed pts $\langle x^*, y^* \rangle$

$\langle 0,0 \rangle, \langle -1,0 \rangle, \langle 1,0 \rangle$

$$\dot{x} = x^3 - x = f(x)$$
$$\dot{y} = -2y = g(y)$$

$$A = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix} = \begin{bmatrix} 3x^2-1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$A_{\langle 0,0\rangle} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$A_{\langle -1,0\rangle} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} \qquad A_{[+1,0]} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

<u>Find eigenvalues of A?</u>

$$\det[A - I\lambda] = \begin{pmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda \end{pmatrix} = 0$$

find λ.

$$(a_{11}-\lambda)(a_{22}-\lambda) - a_{12}a_{21} = 0$$

proceed.... with quadratic

If

$$A = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \text{ then } \lambda \Rightarrow \begin{matrix} a_{11} \\ a_{22} \end{matrix}$$

B/c the characteristic eqn is,

$$(a_{11}-\lambda)(a_{22}-\lambda) = 0$$

$$A_{00} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \quad \lambda \Rightarrow \begin{matrix} -1 \\ -2 \end{matrix}$$ both negatives

Stable node!

$$A_{\pm 1 0} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} \quad \lambda \Rightarrow \begin{matrix} 2 \\ -2 \end{matrix}$$ opp sign!

Saddle!

if

$$A_{x^*, y^*} = \begin{bmatrix} +|a| & 0 \\ 0 & +|b| \end{bmatrix}$$ then unstable node!