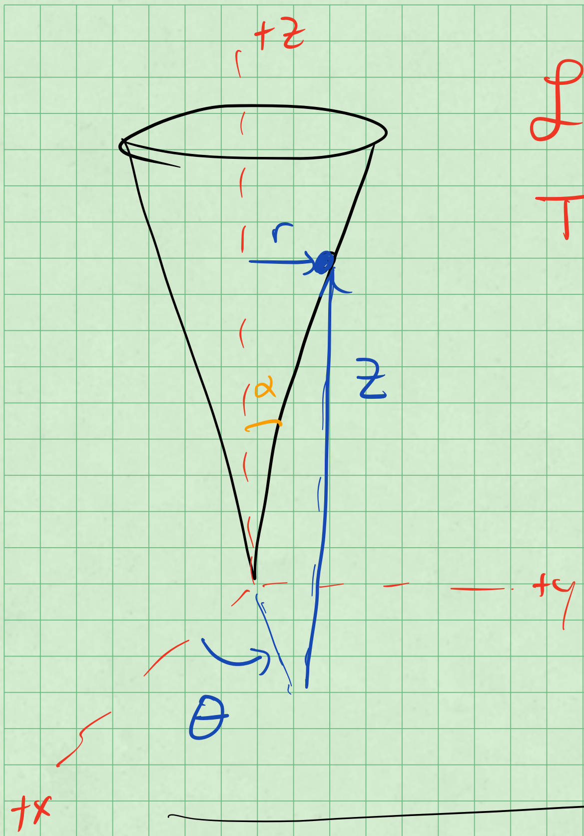
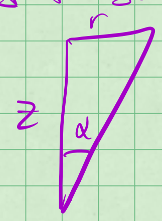


$$\mathcal{L} = T - U \qquad U = mgz$$

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

constrained to surface so that,



$\tan\alpha = \frac{r}{z}$

"constraint eqn"

or $f(r, z) = 0 = r - z\tan\alpha$

① move to cylindrical coords. v^2 in cylindrical (can just look up)

$$U = mgz \qquad T = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2)$$

$$\mathcal{L} = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2) - mgz$$

② put in constraint $z = r\cot\alpha$ note: $\cot\alpha = \text{constant}.$

$$\dot{z} = \dot{r}\cot\alpha$$

$$\mathcal{L} = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{r}^2\cot^2\alpha) - mgr\cot\alpha$$

$$= \frac{1}{2}m(\dot{r}^2[1 + \cot^2\alpha] + r^2\dot{\theta}^2) - mgr\cot\alpha$$

$$\boxed{\mathcal{L} = \frac{1}{2}m\dot{r}^2\csc^2\alpha + \frac{1}{2}mr^2\dot{\theta}^2 - mgr\cot\alpha}$$

two coordinates r & θ

r EOM: $\dfrac{d\mathcal{L}}{dr} - \dfrac{d}{dt}\left(\dfrac{d\mathcal{L}}{d\dot{r}}\right) = 0$

$$mr\dot{\theta}^2 - mg\cot\alpha - \frac{d}{dt}\left(m\dot{r}\csc^2\alpha\right) = 0$$

$$\boxed{m\ddot{r} = \left(mr\dot{\theta}^2 - mg\cot\alpha\right)\cos^2\alpha}$$ ⟸ ala Newton 2

forces: centripetal gravitational

θ EOM: $\dfrac{d\mathcal{L}}{d\theta} - \dfrac{d}{dt}\left(\dfrac{d\mathcal{L}}{d\dot{\theta}}\right) = 0$

$$\frac{d}{dt}\left(mr^2\dot{\theta}\right) = 0 \qquad \underbrace{mr^2\dot{\theta}}_{\text{what is this?}} = \text{constant!}$$

$$\vec{L} = \vec{r} \times \vec{p}$$

Angular Momentum in z!

$$|\vec{L}| = |\vec{r}||\vec{p}|\sin\theta_{rp} = rmv\sin\theta_{rv}$$

$mr^2\dot{\theta} = mrv$ where $v = r\omega$ the linear speed along the plane