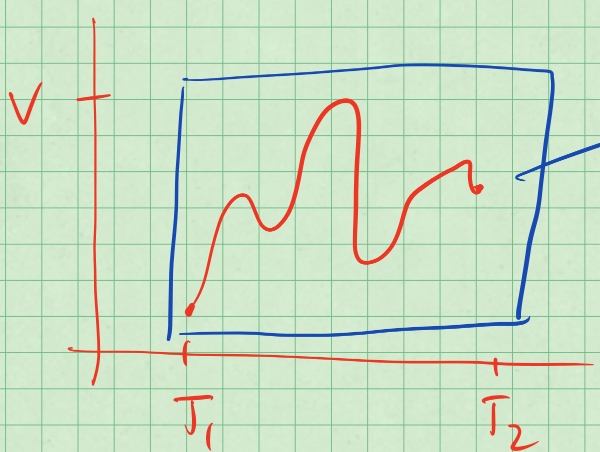
In our prior studies, we assumed the V(t) was continuous (and TBH infinite in extent).

$$f(t) \simeq \sum_{k=0}^{\infty} \alpha_k \cos\left(\frac{2\pi k t}{T_0}\right) + \sum_{k=1}^{\infty} \beta_k \sin\left(\frac{2\pi k t}{T_0}\right)$$

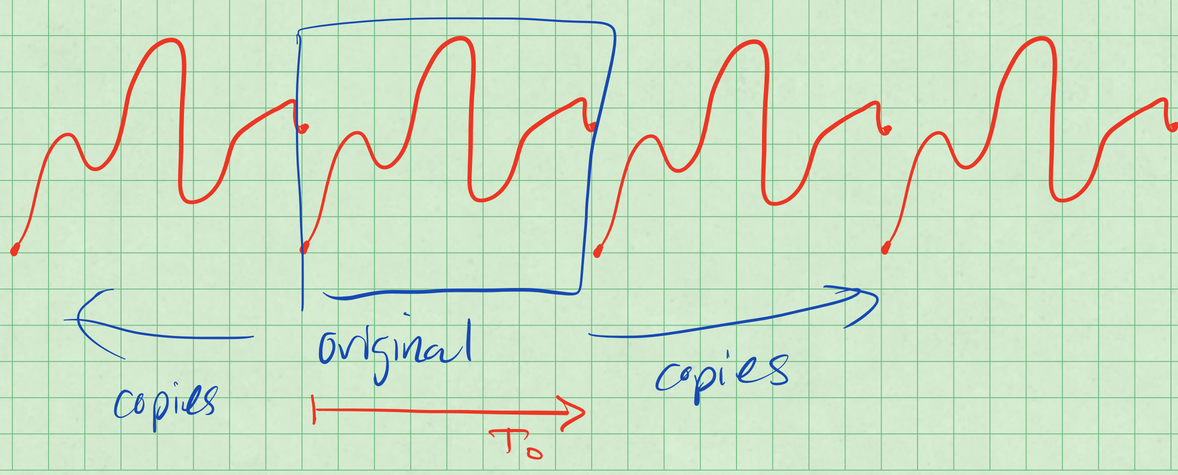
But in science we get



your signal

not infinite and not periodic?

But what if it was?



this is precisely how an FFT approaches a finite signal. We can do the same analytically treating the total signal time $T_2 - T_1$ as the period.

Both analytical and numerical routines cast this problem in the complex domain,

$$f(t) = \sum_{k=-\infty}^{\infty} \gamma_k \, e^{i 2\pi k t / T_0}$$

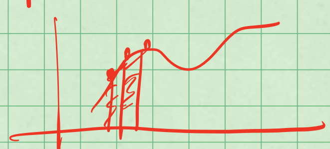
Thus we must find the γ_k's,

$$\underbrace{\gamma_k = \frac{1}{T_0} \int_0^{T_0} V(t) \, e^{-i 2\pi k t / T_0} \, dt}$$

Sometimes this integral is analytic, mostly not.

Enter DFT discrete Fourier Transform

Trapezoidal Rule (Simplest Integrator)



N slices

$$\gamma_k = \frac{1}{L}\frac{1}{N}\left[\frac{1}{2}f(0) + \frac{1}{2}f(T_0) + \sum_{n=1}^{N-1} f(t_n)\, e^{-i 2\pi k t_n / T_0}\right]$$

$$t_n = {}^{n}/_{N}\, T_0$$

$$\gamma_k = \frac{1}{N}\sum_{n=0}^{N-1} \underbrace{f(x_n)}\, e^{-i\, 2\pi k t_n / T_0}$$

y_n just the data!

So our FFT is just solving this integral.