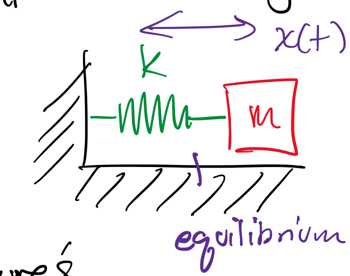
Notes: SHO

Tuesday, September 6, 2022 13:49

The SHO mathematical model is given by

$$\ddot{x} = -\frac{k}{m}x$$



As we have seen there's several general solution forms. these simply satisfy the ODE. they are not a particular solution, which requires the initial conditions.

Finding the particular solution

Given the general solution to an ODE, we simply need to find the undetermined coefficients by plugging in those conditions called and

Ex: $x(t=0)=x_0$ $\qquad$ $\dot{x}(t=0)=0 \Leftarrow$ released

$$x(t)=A\cos(\omega t)+B\sin(\omega t)$$

$$x(t=0)=x_0=A\underbrace{\cos(0)}_{1}+B\underbrace{\sin(0)}_{0}$$

we get no info about B

$$\boxed{A=x_0}$$

$$x(t)=x_0\cos(\omega t)+B\sin(\omega t)$$

$$\dot{x}(t)=-\omega x_0\sin(\omega t)+\omega B\cos(\omega t)$$

cool we get B!

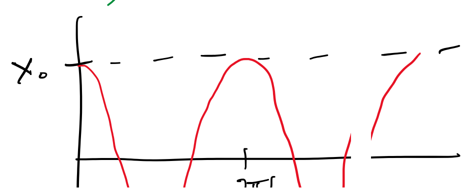
$$\dot{x}(t=0)=0=-\omega x_0\underbrace{\sin(0)}_{0}+\omega B\underbrace{\cos(0)}_{1}$$

$$\omega B=0 \Rightarrow \boxed{B=0}$$ b/c $\omega \neq 0$ by defn.

Thus the particular solution for

$x(0)=x_0$ $\qquad$ $\dot{x}(0)=0$ is,

$$\boxed{x(t)=x_0\cos(\omega t)}$$



Ex: Let's try a different solution form

$$x(t) = \underbrace{C}\sin(\omega t + \underbrace{\phi})$$ ⇐ still a general solution

2 free parameters still

$$x(0) = x_0 = C\sin(0+\phi) = C\sin(\phi)$$

$$\boxed{C\sin(\phi) = x_0}$$

$$\dot{x}(t) = \omega C\cos(\omega t + \phi)$$

$$\dot{x}(0) = 0 = \underbrace{\omega}_{\text{nonzero so}} C\cos(\phi)$$

$$\boxed{C\cos(\phi) = 0}$$

We have 2 equations,

① $C\sin\phi = x_0$ ② $C\cos\phi = 0$

Method 1:

Dividing by zero is bad so lets divide (2) by (1),

$$\frac{C\cos\phi}{C\sin\phi} = \frac{0}{x_0} = 0 = \cot\phi$$

b/c $\cot\phi = \dfrac{1}{\tan\phi}$ $\quad \cot\phi \to 0$

Where does tangent blow up? $\tan\phi \to \pm\infty$

$\phi = \pm \pi/2,\ \pm\frac{3\pi}{2}$, etc.

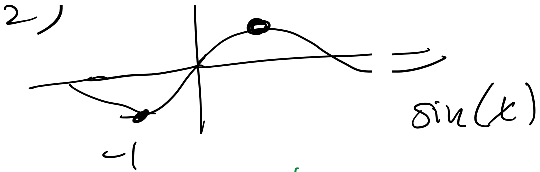
We can represent any phase shift $> \pi$ with a negative shift, so really we have

$\phi = \pm \pi/2$ as possible solutions

[illegible]

$x(t) = C\sin(\omega t - \pi/2)$

$x(0) = \underbrace{C\sin(\pm\pi/2)}_{\pm 1\ !!!}$ $\Leftarrow$ oh! if we choose $\pm\pi/2$ we change sign!



Pick! and the sign of C is known.

option 1: $\phi = +\pi/2$

$x(0) = C\sin(\pi/2) = C = x_0$

option 2: $\phi = -\pi/2$

$x(0) = C\sin(-\pi/2) = -C = x_0$

Two valid solutions

$x(t) = C\sin(\omega t + \pi/2)$

$x(t) = -C\sin(\omega t - \pi/2)$

Both work and are related.

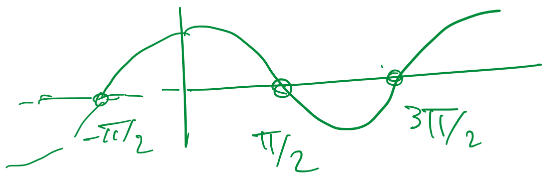
Method 2:

Look at (2) $\quad C\cos\phi = 0$

$C \neq 0$ otherwise no amplitude $\Rightarrow$ no oscillation!

So!

$\cos\phi = 0$



$\phi = \pm \pi/2, \pm 3\pi/2$, etc.

Same arguments give us $\phi = \pm \pi/2$

and same solutions,

$$x(t) = C \sin(\omega t + \pi/2)$$

$$x(t) = -C \sin(\omega t - \pi/2)$$