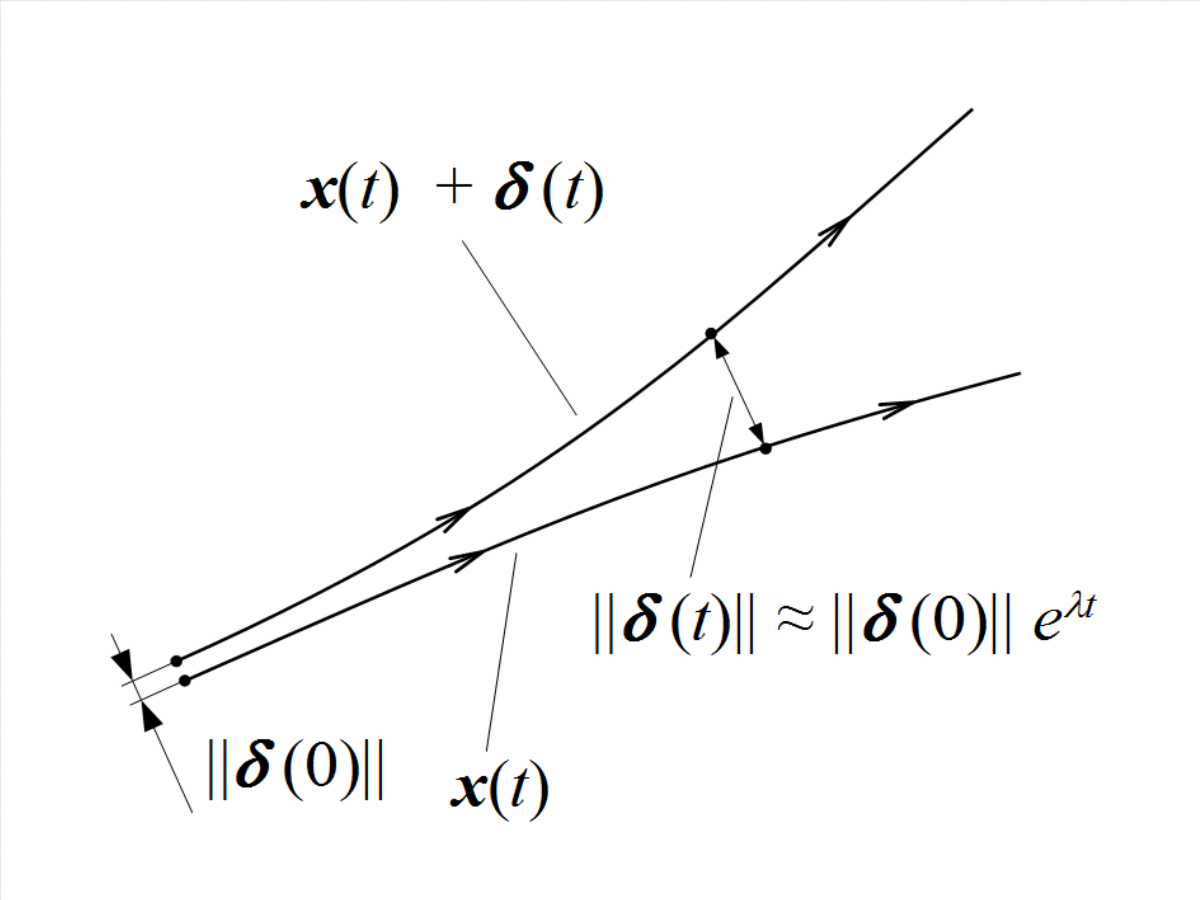
Day 28 - Hallmarks of Chaos

Conceptualizing the Lyapunov Exponent

Trajectories diverge exponentially in time



Announcements

- Midterm 1 is graded
- Homework 7 is due Friday
  - No homework next week
- Midterm 2 will be assigned next Monday (due 18 April)
  - Second project check-in

Seminars This Week

WEDNESDAY, March 26, 2025

- **Astronomy Seminar**, 1:30 pm, 1400 BPS, **Bryan Terrazas, Oberlin College**, *Galaxy evolution and feedback modeling*
- FRIB Nuclear Science Seminar, 3:30pm., FRIB 1300 Auditorium, **Dr. Jacklyn Gates of Lawrence Berkeley National Laboratory**, *Toward Pursuing New Superheavy Elements*

Seminars This Week

THURSDAY, March 27, 2025

- **Special FRIB/MSU Nuclear Science Seminar with Colloquium**, 3:30 pm, 1415 BPS, **Mandie Gehring, LANL**, *Measuring Intense X-ray Spectra and an Overview of Space Research at Los Alamos National Laboratory*

FRIDAY, March 28, 2025

- **IReNA Online Seminar**, 2:00 pm, In Person and Zoom, FRIB 2025 Nuclear Conference Room, **Jordi José, Technical University of Catalonia, UPC (Barcelona, Spain)**, *Classical novae at the crossroads of nuclear physics, astrophysics and cosmochemistry*

Hallmarks of a Classically Chaotic System

1. **Deterministic**
2. **Sensitive to Initial Conditions**
3. **Non-periodic Behavior**
4. **Strange Attractors**
5. **Parameter Sensitivity**
6. (Sometimes) **Periodic Behavior**

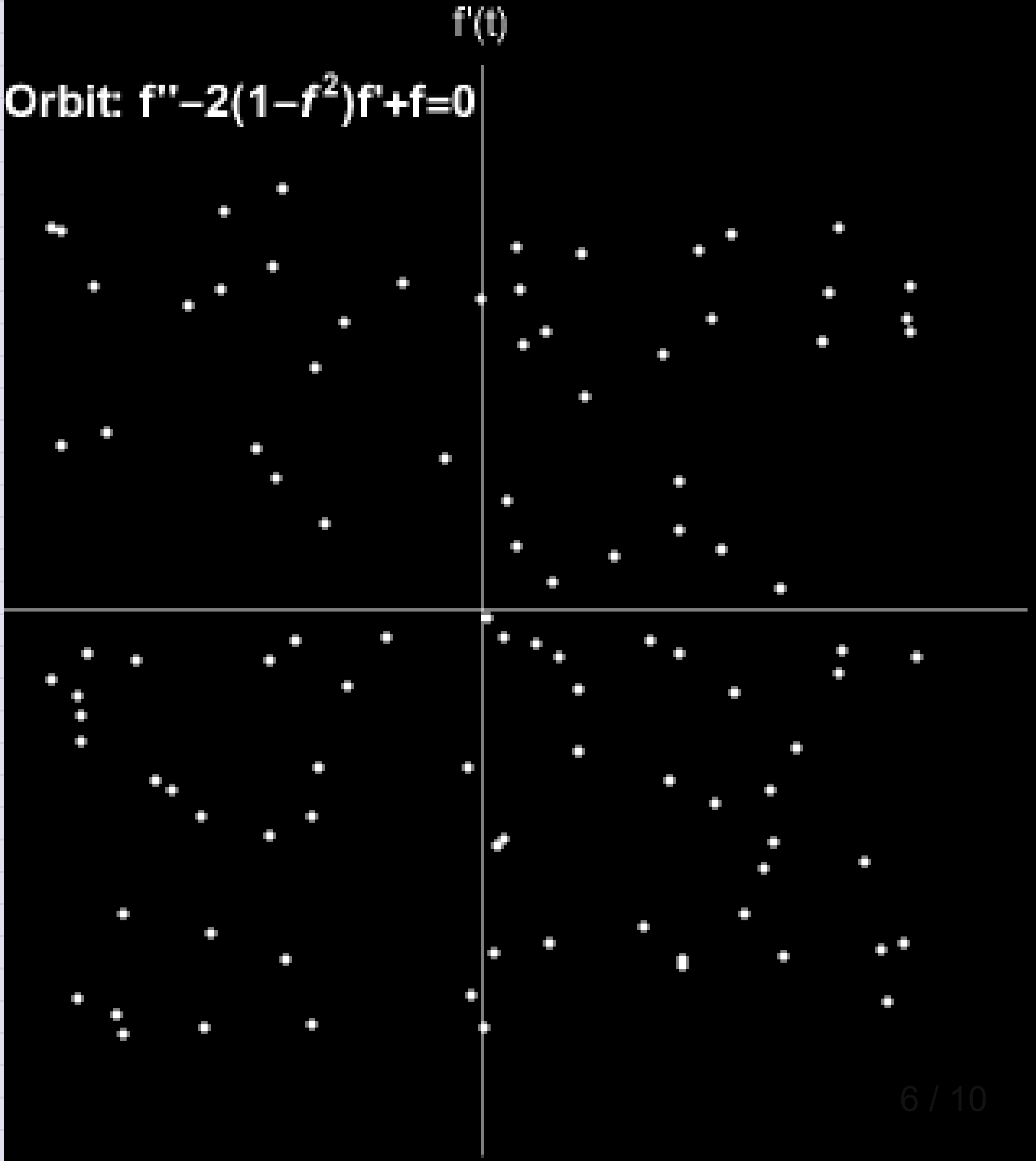
Limit Cycle

A **limit cycle** is a closed trajectory in phase space that is an attractor for a dynamical system.

The **Van der Pol Oscillator** exhibits a limit cycle.

$$\ddot{x} - \mu(1 - x^2)\dot{x} + x = 0$$

Random initial conditions converge to a limit cycle. Modeled with $\mu = 2$.



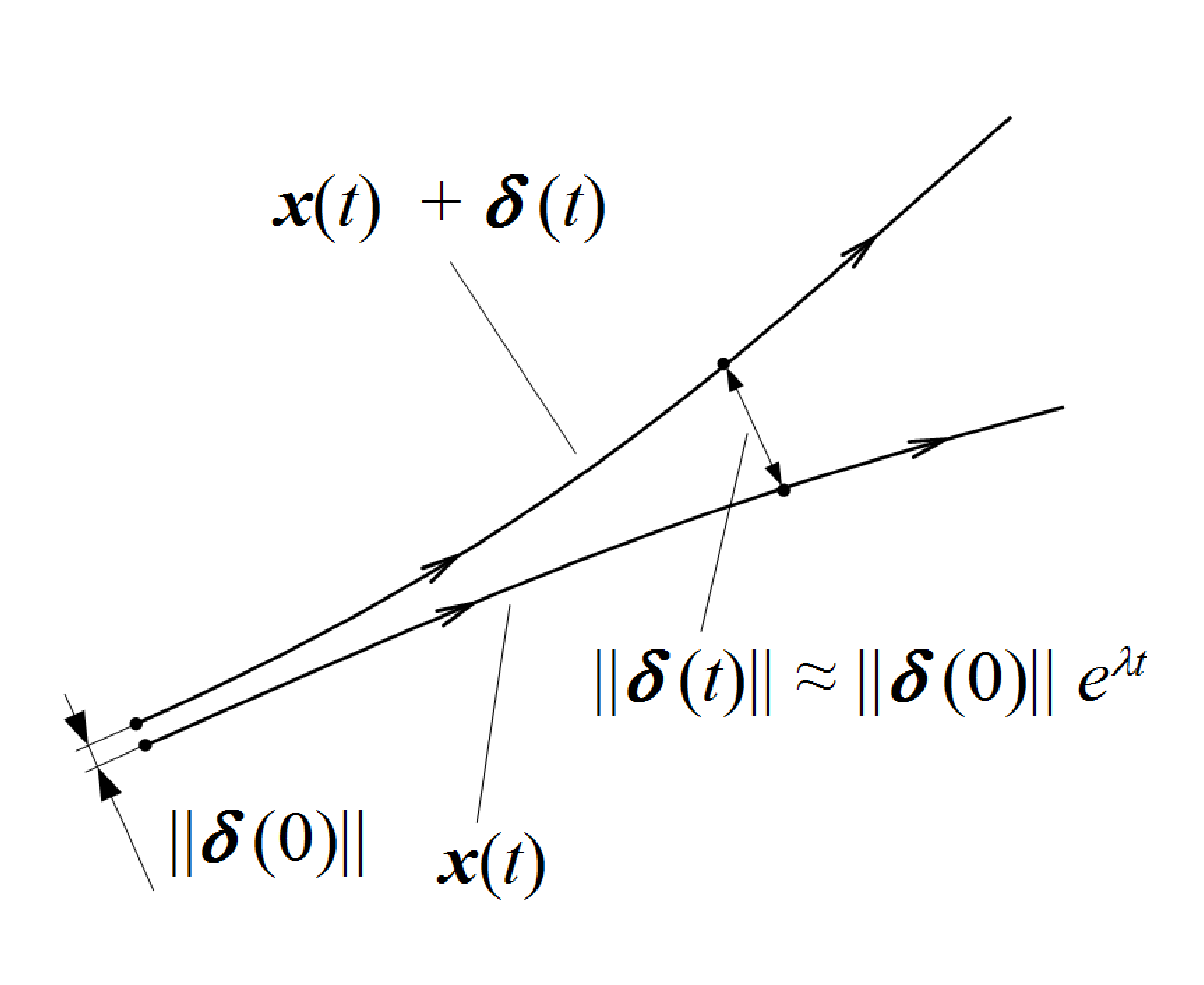
The Lyapunov Exponent

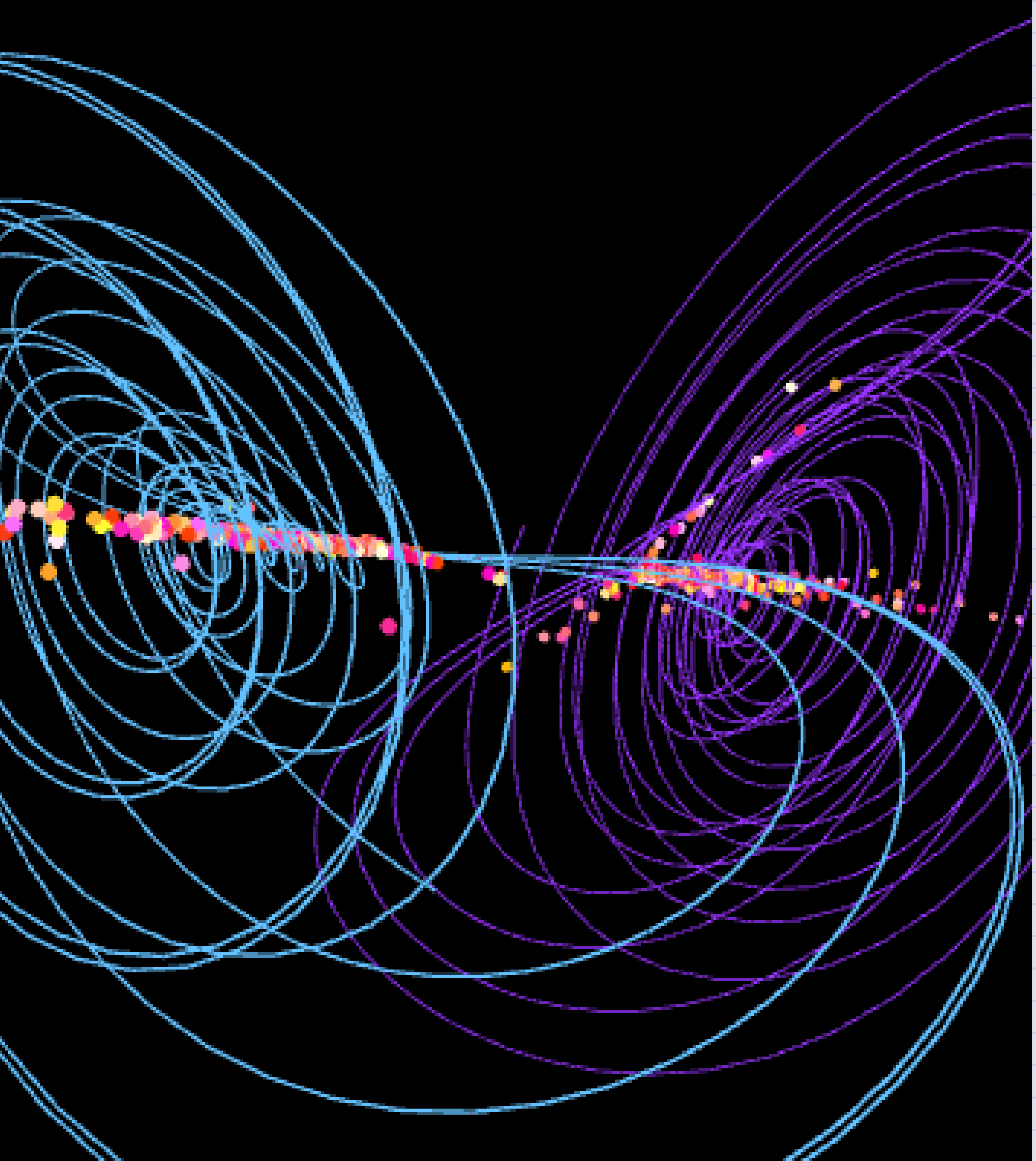
$\vec{\delta}(t)$ is the separation vector between two trajectories in phase space $\vec{\delta}(t) = \vec{x}_2(t) - \vec{x}_1(t)$.

Do trajectories diverge exponentially in time, $|\vec{\delta}(t)| \approx |\vec{\delta}(0)|e^{\lambda t}$?

Each phase coordinate can change at a different rate: $\vec{\lambda} = \langle \lambda_1, \lambda_2, \ldots, \lambda_n \rangle$.

Largest $\lambda_i > 0$? Chaotic system.





Strange Attractors

A **strange attractor** is a set of points in phase space that a chaotic system approaches.

Chen Attractor

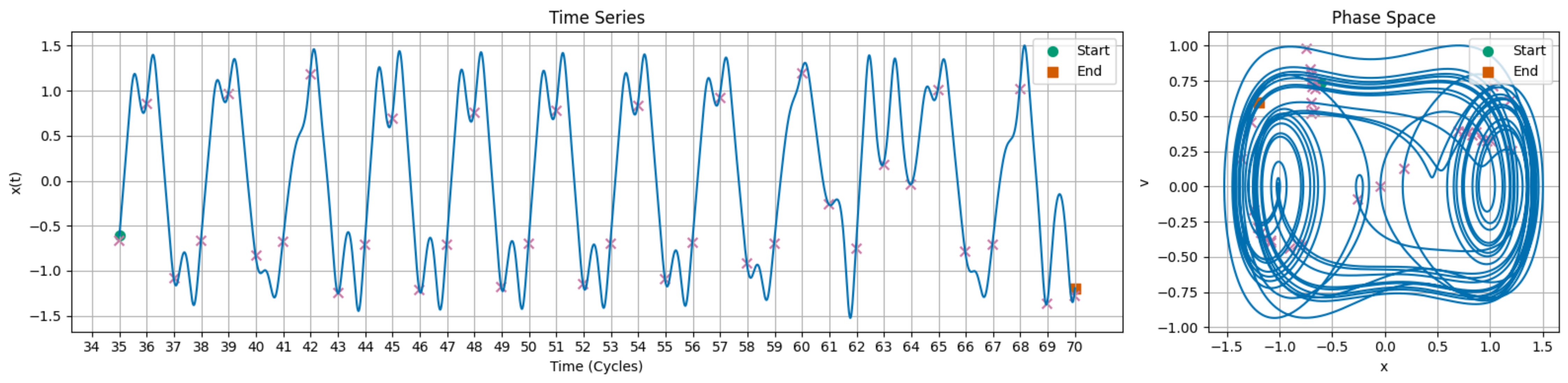
$$\dot{x} = \alpha x - yz$$
$$\dot{y} = \beta y + xz$$
$$\dot{z} = \gamma z + xy/3$$

$\alpha = 5, \beta = -10, \gamma = -0.38.$

Interactive 3D Model

Example 1: Duffing Equation

$$\ddot{x} + \beta\dot{x} + \alpha x + \gamma x^3 = F_0 \cos(\omega t)$$



Exhibits Periodic and Chaotic Behavior

Illustrates period doubling bifurcations as route to chaos

Example 2: Lorenz System

$$\dot{x} = \sigma(y - x)$$
$$\dot{y} = x(\rho - z) - y$$
$$\dot{z} = xy - \beta z$$

Exhibits sensitive dependence on initial conditions

Demonstrates the concept of a strange attractor