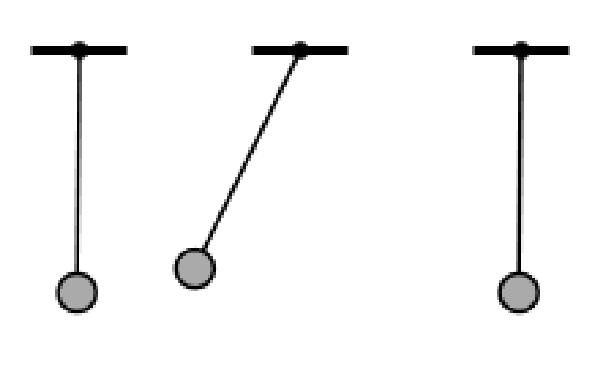
Day 24 - Driven Oscillations

Resonance in a driven pendulum system. ⟶

Source: Wikipedia



Announcements

- Midterm 1 is still being graded
- Homework 6 is due next Friday
- Homework 7 is posted, due next Friday
  - Final project check-in #1
- Danny will be out this Wednesday
  - Class will be on zoom https://msu.zoom.us/j/92295821308
  - passcode: phy321
  - No office hours Wednesday

Seminars this week

Most of MSU folks are at APS Global Physics Summit

WEDNESDAY, March 19, 2025

- Astronomy Seminar, 1:30 pm, 1400 BPS, Alex Rodriguez, University of Michigan, *Galaxy clusters, cosmology, and velocity dispersion*
- FRIB Nuclear Science Seminar, 3:30pm., FRIB 1300 Auditorium, Dr. Pierre Morfouace of CEA-DAM, *Mapping the new asymmetric fission island with the R3B/SOFIA setup*

THURSDAY, March 20, 2025

Colloquium, 3:30 pm, 1415 BPS, Guillaume Pignol, University of Grenoble, *Ultracold neutrons: a precision tool in fundamental physics*

Reminders

We solved the damped harmonic oscillator equation:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$$

We chose a solution (**ansatz**) of the form

$$x(t) = C_1 e^{rt} + C_2 e^{rt}$$

and computed the roots of the characteristic equation:

$$r^2 + 2\beta r + \omega_0^2 = 0$$

We found the roots to be:

$$r = -\beta \pm \sqrt{\beta^2 - \omega_0^2}$$

Weak Damping

We found that when $\beta^2 < \omega_0^2$, the roots are complex:

$$r = -\beta \pm i\sqrt{\omega_0^2 - \beta^2}$$

This means that the solution is oscillatory:

$$x(t) = e^{-\beta t}\left(C_1 \cos(\sqrt{\omega_0^2 - \beta^2}t) + C_2 \sin(\sqrt{\omega_0^2 - \beta^2}t)\right)$$

The solution is a damped oscillation with frequency $\omega_1 = \sqrt{\omega_0^2 - \beta^2}$.

Strong Damping

When $\beta^2 > \omega_0^2$, the roots are real:

$$r = -\beta \pm \sqrt{\beta^2 - \omega_0^2}$$

This means that the solution is not oscillatory:

$$x(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$$

where $r_1 = -\beta + \sqrt{\beta^2 - \omega_0^2} < 0$ and $r_2 = -\beta - \sqrt{\beta^2 - \omega_0^2} < 0$.

The solution is the sum of two exponentials with different decay rates.

Critical Damping

When $\beta^2 = \omega_0^2$, the roots are real and equal (repeated roots):

$$r = -\beta$$

This means that the solution is not oscillatory, but also that our ansatz is not sufficient. The correct form of the solution is:

$$x(t) = (C_1 + C_2 t)e^{-\beta t}$$

In most cases, we will work with weak damping.

Clicker Question 24-1

Next week, we can choose what to do in class. What would you like to do?

1. Fourier Series for solving ODEs (currently scheduled)
2. Pathways to Chaos (build off oscillations)
3. Collisions and Conservation Laws (momentum and angular momentum)
4. Calculus of Variations (spend more time on this?)
5. Other ideas?

Clicker Question 24-2

What do we expect the phase space diagram (x vs $\dot{x}$) to look like for a weakly damped harmonic oscillator?

1. A set of ellipses
2. A set of spirals
3. Depends on how weak the damping is
4. Depends on the total energy
5. More than one of the above

Clicker Question 24-3

The driven harmonic oscillator equation is:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f(t)$$

with $w_0^2 = k/m$ and $2\beta = b/m$. What is the dimension of the driving force $f(t)$?

1. Force (Newtons, N)
2. Force per unit second (N/s)
3. Force per unit length (N/m)
4. Force per unit mass (N/kg)

Clicker Question 24-4

The driven harmonic oscillator equation is:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f(t).$$

This ODE is a _________ differential equation.

1. linear
2. nonlinear
3. first-order
4. second-order
5. more than one of the above

Example: Sinusoidal Driving Force

Let $f(t) = f_0 \cos(\omega t)$, so that the driven harmonic oscillator equation is:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f_0 \cos(\omega t)$$

Note: $\omega \neq \omega_0$

Note that if the driving follows a sine wave, then we have:

$$\ddot{y} + 2\beta\dot{y} + \omega_0^2 y = f_0 \sin(\omega t)$$

Interesting, $e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$, let try to work with $z(t) = x(t) + iy(t)$.

Clicker Question 24-5

We found that the square amplitude of the driven harmonic oscillator is:

$$A^2 = \frac{f_0^2}{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}$$

When is the amplitude of the driven oscillator maximized?

1. When the driving frequency (ω) is far from the natural frequency (ω_0)
2. When the driving frequency (ω) is close to the natural frequency (ω_0)
3. When the damping (2β) is weak
4. When the damping (2β) is strong
5. Some combination of the above