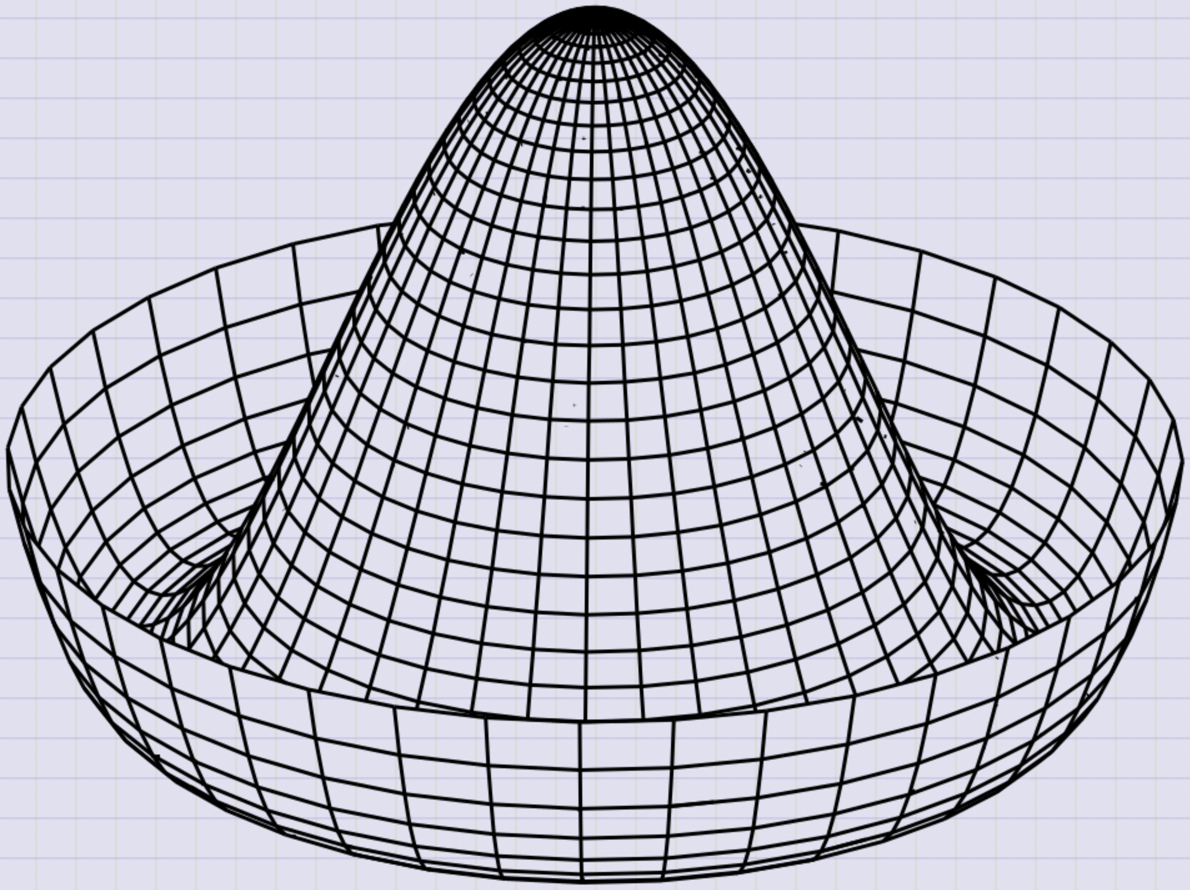
Day 15 - Potential Energy and Stability

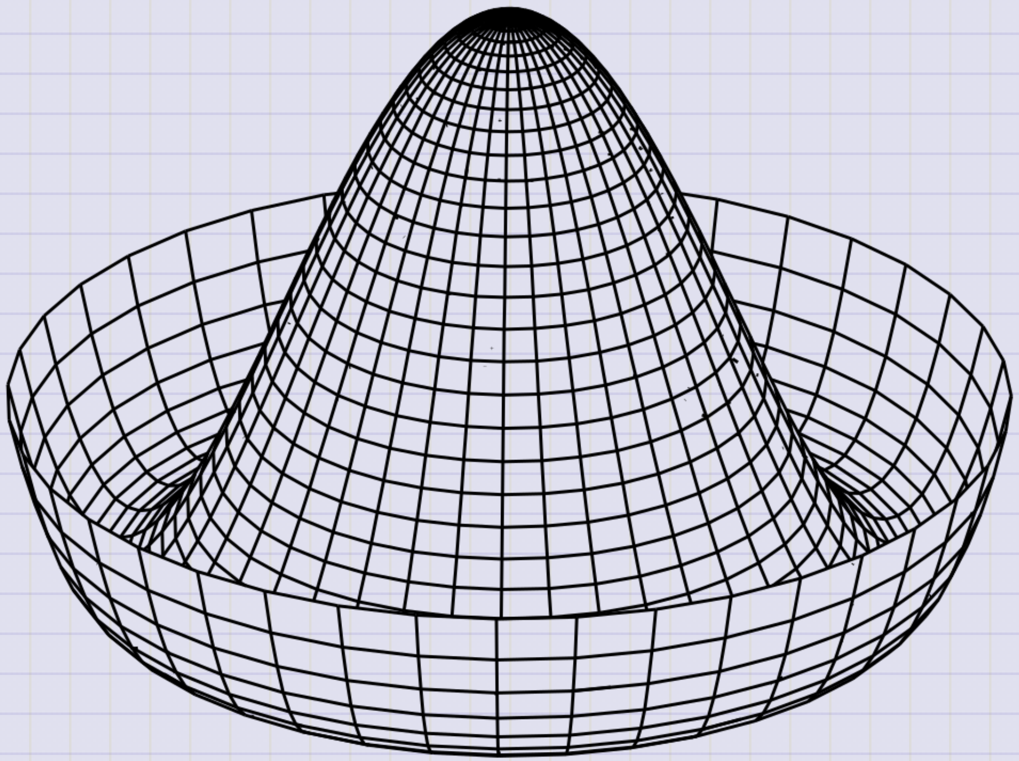
Mexican Hat/Sombrero Potential ⟶



Mexican Hat Potential

$$V(\phi) = -5|\phi|^2 + |\phi|^4$$

- Spontaneous Symmetry Breaking (Jeffery Goldstone, 1961)
- Unstable vacuum state at $\phi = 0$
  - Peak of the hat
- Infinite number of stable minima
  - $\phi = \sqrt{5/2}e^{i\phi}$



Announcements

- HW 4 is due today
- Midterm 1 is available today (Due Feb 24th)

Seminars this week

MONDAY, February 17, 2025

- QuIC Seminar, 12:30 pm, 1400 BPS, Dr. Michael Hilke, *The history of quantum computing*
- High Energy Physics Seminar, 1:30pm, BPS 1400 BPS, Joshua Isaacson, *Event Generation for Next-Gen HEP Experiments*
- Condensed Matter Seminar 4:10 pm, 1400 BPS, Lisa Lapidus, *The Physics of Biomolecular Condensation*

Seminars this week

TUESDAY, February 18, 2025

- Theory Seminar, 11:00am., FRIB 1200 lab, Ibrahim Abdurahman, *Investigating Fission Dynamics within Time-Dependent Density Functional Theory Extended to Superfluid Systems*

Seminars this week

WEDNESDAY, February 19, 2025

- Astronomy Seminar, 1:30 pm, 1400 BPS, Aaron Bello-Arufa, *The atmospheres of small exoplanets with JWST*
- **PER Seminar**, 3:00 pm., BPS 1400, Anthony Escuardo, *OPTYCS: A Community of Practice Supporting Teaching and Scholarship at Two-Year College*
- FRIB Nuclear Science Seminar, 3:30pm., FRIB 1300 Auditorium, Elise Novitski, *A new approach to measuring neutrino mass*

Seminars this week

THURSDAY, February 20, 2025

- High Energy Physics Seminar, 1:30pm, BPS 1400 BPS, Ben Assi, *Precision QCD and EFT for Next-Generation Collider Studies*
- Physics and Astronomy sColloquium, 3:30 pm, 1415 BPS, Eric Hudson, *Laser spectroscopy of a nucleus*

This Week's Goals

- Understand the concept of potential energy
- Determine the equilibrium points of a system using potential energy
- Analyze the stability of equilibrium points
- Define and begin to apply conservation of linear and angular momentum

Reminders: Conservative Forces

- Conservative forces are those with zero curl

$$\nabla \times \vec{F} = 0$$

- The work done by a conservative force is path-independent; on a closed path, the work done is zero

$$\oint \vec{F} \cdot d\vec{r} = 0$$

- The force can be written as the gradient of a scalar potential energy function

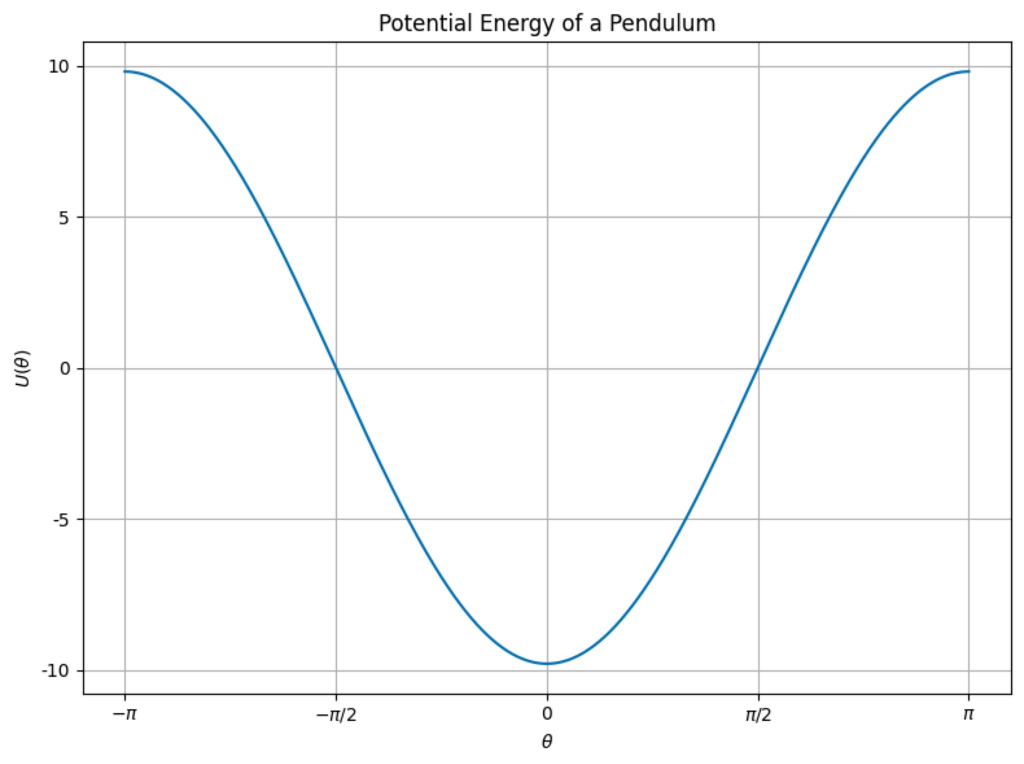
$$\vec{F} = -\nabla U$$

Clicker Question 15-1

Here's the graph of the potential energy function $U(x)$ for a pendulum.

What can you say about the equilibrium points? There is/are:

1. One stable point
2. Two stable points
3. One stable and one unstable point
4. Two unstable and one stable point



Clicker Question 15-2

Here's a potential energy function $U(x)$ for a pendulum:

$$U(\phi) = -mgL\cos(\phi) + U_0$$

1. Find the equilibrium points (ϕ^*) of the pendulum by setting:

$$\frac{dU(\phi^*)}{d\phi} = 0.$$

2. Characterize the stability of the equilibrium points (ϕ^*) by examining the second derivative:

$$\frac{d^2U(\phi^*)}{d\phi^2} > 0? \qquad \frac{d^2U(\phi^*)}{d\phi^2} < 0?$$

Click when done.

Clicker Question 15-3

A double-well potential energy function $U(x)$ is given by

$$U(x) = -\frac{1}{2}kx^2 + \frac{1}{4}kx^4.$$

We assume we have scaled the potential energy so that all the units are consistent.

How many equilibrium points does this system have?

1. 1
2. 2
3. 3
4. 4

Clicker Question 15-4

A double-well potential energy function $U(x)$ is given by

$$U(x) = -\frac{1}{2}kx^2 + \frac{1}{4}kx^4.$$

1. Find the equilibrium points (x^*) of the pendulum by setting:

$$\frac{dU(x^*)}{dx} = 0.$$

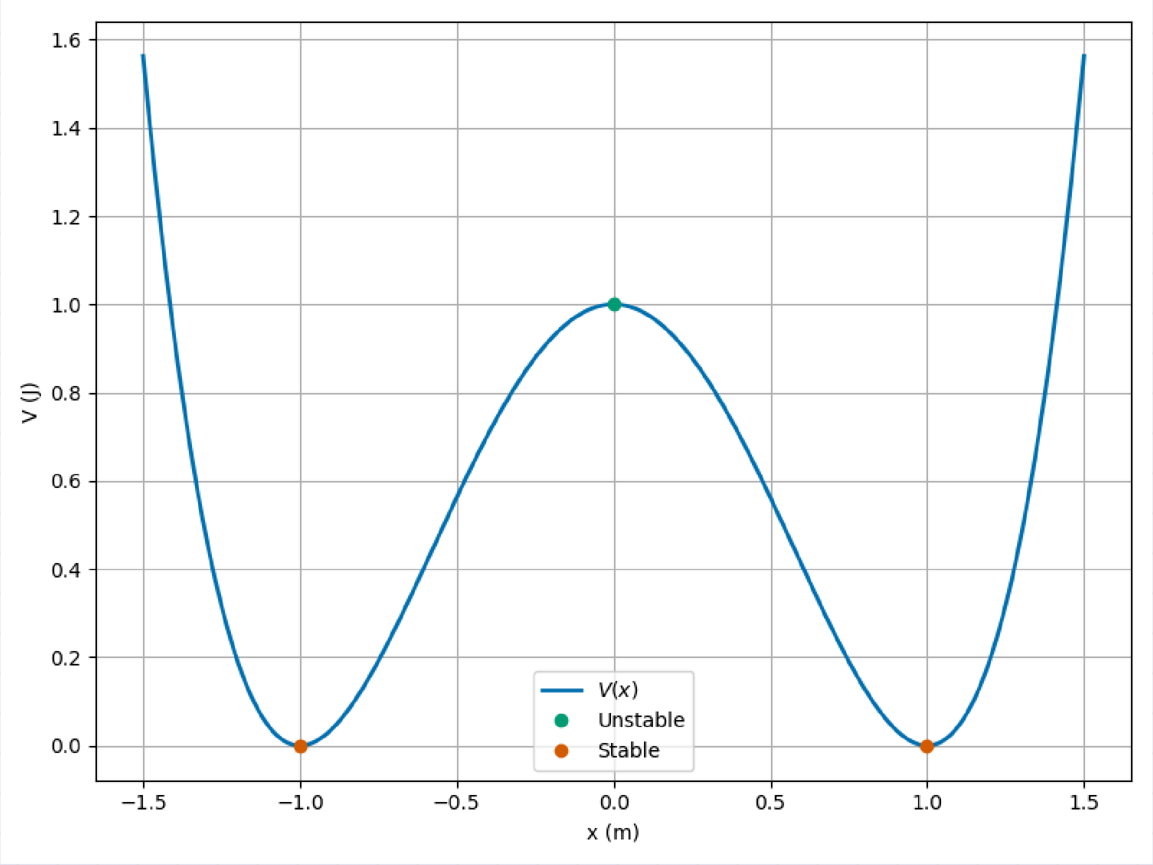
2. Characterize the stability of the equilibrium points (x^*) by examining the second derivative:

$$\frac{d^2U(x^*)}{dx^2} > 0? \qquad \frac{d^2U(x^*)}{dx^2} < 0?$$

Click when done.

Clicker Question 15-5

Here's a graph of the potential energy function $U(x)$ for a double-well potential.



Describe the motion of a particle with the total energy, $E =$

1. $0.4\,\mathrm{J}$, $<$ barrier height
2. $1.2\,\mathrm{J}$, $>$ barrier height
3. $1.0\,\mathrm{J}$, $=$ barrier height

Click when done.