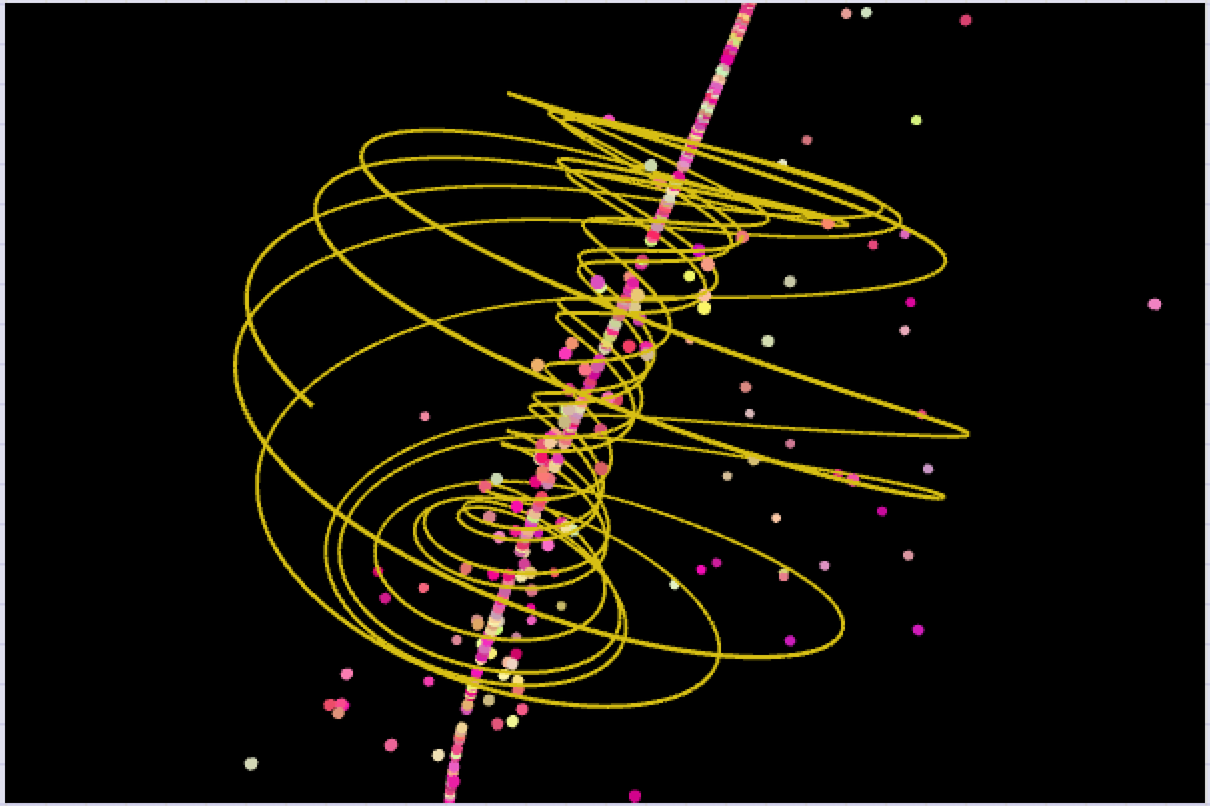
Day 27 - Introduction to Chaos

Sprott Attractor

$$\dot{x} = y + axy + xz$$
$$\dot{y} = 1 - bx^2 + yz$$
$$\dot{z} = x - x^3 - y^2$$
$$a = 2.07 \quad b = 1.79$$

https://www.dynamicmath.xyz/strange-attractors/



Announcements

- Midterm 1 (problems 2 and 3) is graded
  - Problem 1 is still being graded
- Homework 7 is due Friday
  - No homework next week
- Midterm 2 will be assigned next Monday (due 18 April)
  - Second project check-in

Seminars This Week

MONDAY, March 24, 2025

- **Condensed Matter Seminar** 4:10 pm, 1400 BPS, **Andrew Kirkpatrick, MSU**, *Fabrication of orientated NV centres in diamond by ultrafast laser fabrication* AND **Ankang Liu, MSU** *Effect of hole-strain coupling on the eigenmodes of semiconductor-based nanomechanical systems*

Seminars This Week

WEDNESDAY, March 26, 2025

- **Astronomy Seminar**, 1:30 pm, 1400 BPS, **Bryan Terrazas, Oberlin College**, *Galaxy evolution and feedback modeling*
- FRIB Nuclear Science Seminar, 3:30pm., FRIB 1300 Auditorium, **Dr. Jacklyn Gates of Lawrence Berkeley National Laboratory**, *Toward Pursuing New Superheavy Elements*

Seminars This Week

THURSDAY, March 27, 2025

- **Special FRIB/MSU Nuclear Science Seminar with Colloquium**, 3:30 pm, 1415 BPS, **Mandie Gehring, LANL**, *Measuring Intense X-ray Spectra and an Overview of Space Research at Los Alamos National Laboratory*

FRIDAY, March 28, 2025

- **IReNA Online Seminar**, 2:00 pm, In Person and Zoom, FRIB 2025 Nuclear Conference Room, **Jordi José, Technical University of Catalonia, UPC (Barcelona, Spain)**, *Classical novae at the crossroads of nuclear physics, astrophysics and cosmochemistry*

What is Chaos?

At your table, discuss what it means for a system to be chaotic.

- What are some examples of chaotic systems?
- What are some characteristics of chaotic systems?
- How do chaotic systems differ from non-chaotic systems?

Try to come up with two answers to each question to share.

Hallmarks of a Classically Chaotic System

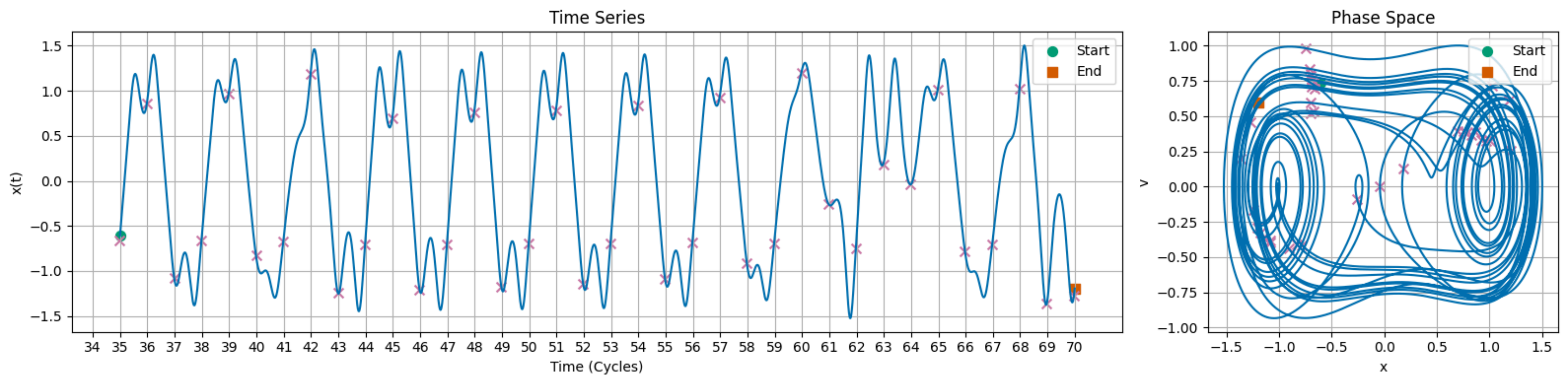
1. **Deterministic**: The system is governed by deterministic laws (e.g., Newton's laws of motion, a set of differential equations)
2. **Sensitive to Initial Conditions**: A bundle of trajectories that start close together will diverge exponentially over time
3. **Non-periodic Behavior**: The system exhibist s complex, non-repeating behavior over time, this might look like "random" behavior, but it is not truly random because it is deterministic

Hallmarks of a Classically Chaotic System

4. **Strange Attractors**: The system may have a strange attractor, which is a fractal structure in phase space that the system tends to evolve towards over time
5. **Parameter Sensitivity**: The system may be sensitive to small changes in parameters, which can trigger qualitative changes in the system's behavior
6. (Sometimes) **Periodic Behavior**: The system may exhibit periodic behavior for certain parameter values, and this might be a signal that the system bifurcates into chaotic behavior for other parameter values

Example 1: Duffing Equation

$$\ddot{x} + \beta\dot{x} + \alpha x + \gamma x^3 = F_0 \cos(\omega t)$$



Exhibits Periodic and Chaotic Behavior

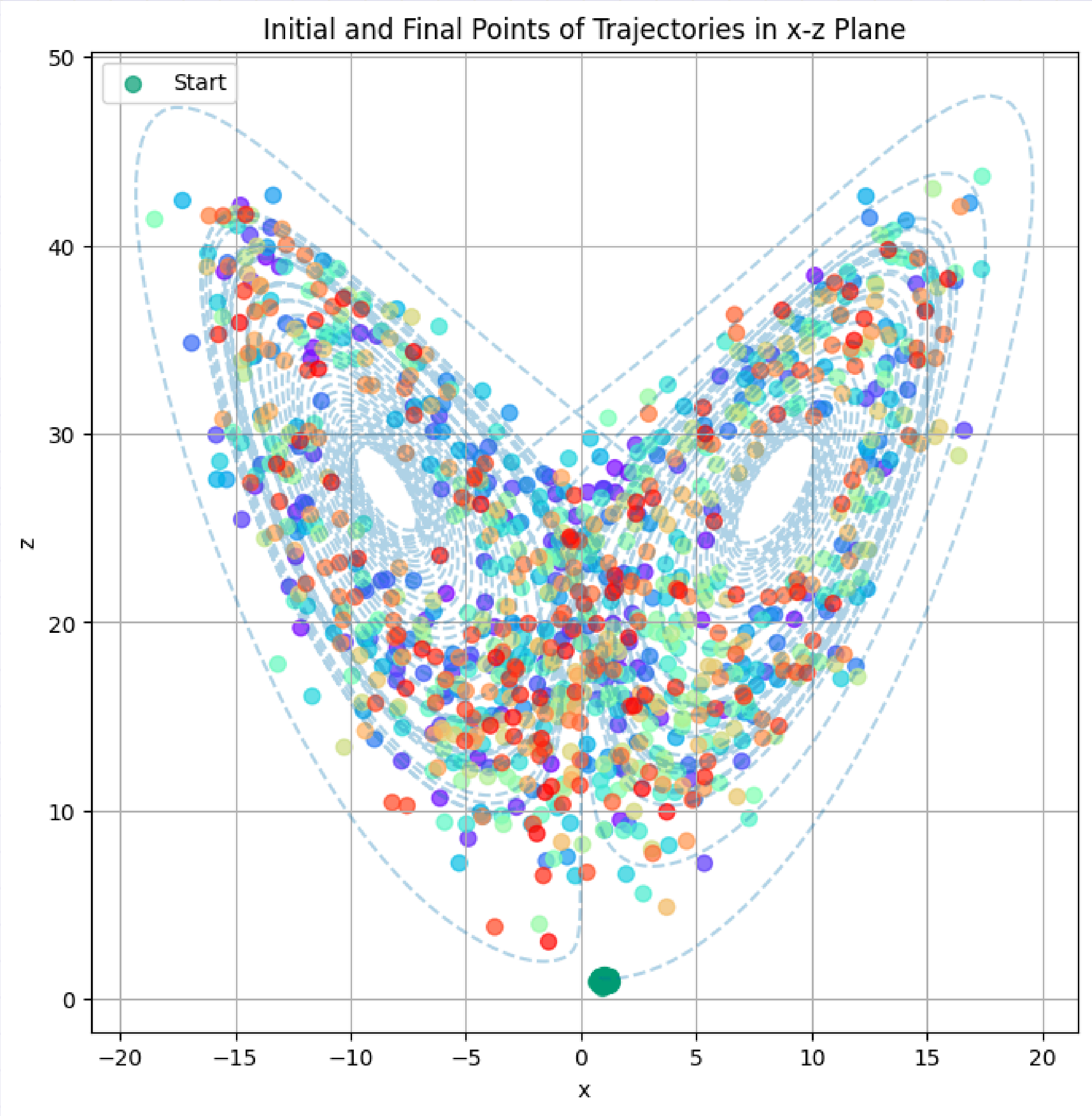
Illustrates period doubling bifurcations as route to chaos

Example 2: Lorenz System

$$\dot{x} = \sigma(y - x)$$
$$\dot{y} = x(\rho - z) - y$$
$$\dot{z} = xy - \beta z$$

Exhibits sensitive dependence on initial conditions

Demonstrates the concept of a strange attractor



Using `solve_ivp`

We will start with the damped driven pendulum as an example. This will illustrate how to use `solve_ivp` to solve a system of coupled first-order differential equations.

$$\ddot{\theta} + \beta\dot{\theta} + \sin(\theta) = A\cos(\omega_D t)$$

We can rewrite this as two first-order equations:

$$\dot{\theta} = \omega$$
$$\dot{\omega} = -\beta\omega - \sin(\theta) + A\cos(\omega_D t)$$

Using `solve_ivp`

To use `solve_ivp`, we write a function for the derivatives:

```python
def damped_driven_pendulum(t, y, beta, A, omegaD=1):
    theta, omega = y
    dtheta_dt = omega
    domega_dt = -np.sin(theta) - beta * omega + A * np.cos(omegaD*t)
    return [dtheta_dt, domega_dt]
```

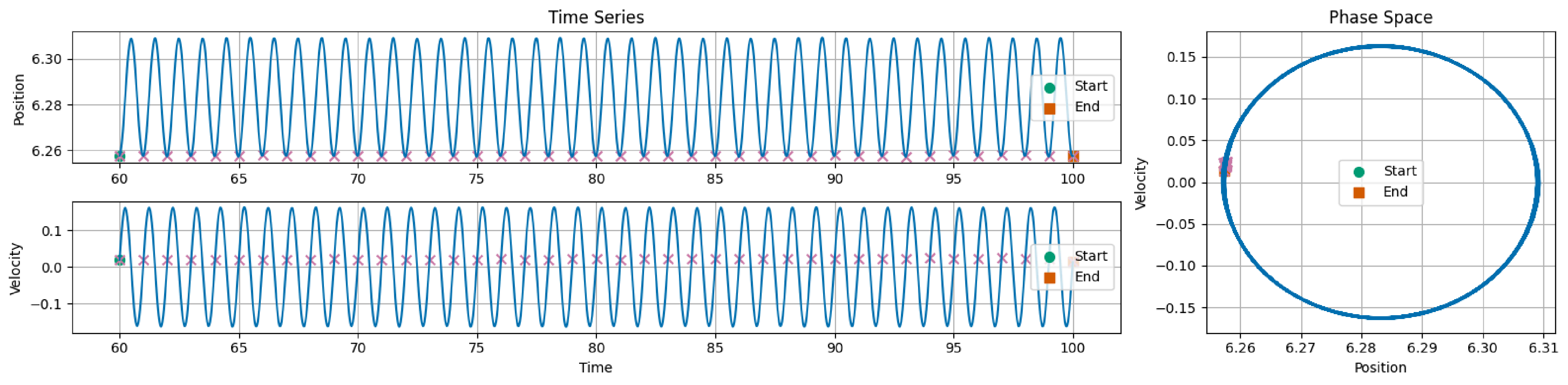
Using `solve_ivp`

Now we can use `solve_ivp` to solve the system of equations:

```
# Parameters that define the system
beta = 0.5
A = 1.0
omegaD = 2*np.pi

# Time span for the simulation
t_span = (0, 100)

# Initial conditions: [theta, omega]
y0 = [6, 0]

# Time points where we want the solution
t_eval = np.linspace(t_span[0], t_span[1], 10000)

# Solve the system of equations
solution = solve_ivp(damped_driven_pendulum, t_span, y0, args=(beta, A, omegaD), t_eval=t_eval)
```

Damped Driven Pendulum

Long Term Behavior is Periodic



"Period-1" Dynamics is a term to indicate there's a single frequency governing the motion

Phase space plots can provide a better window into the system's behavior