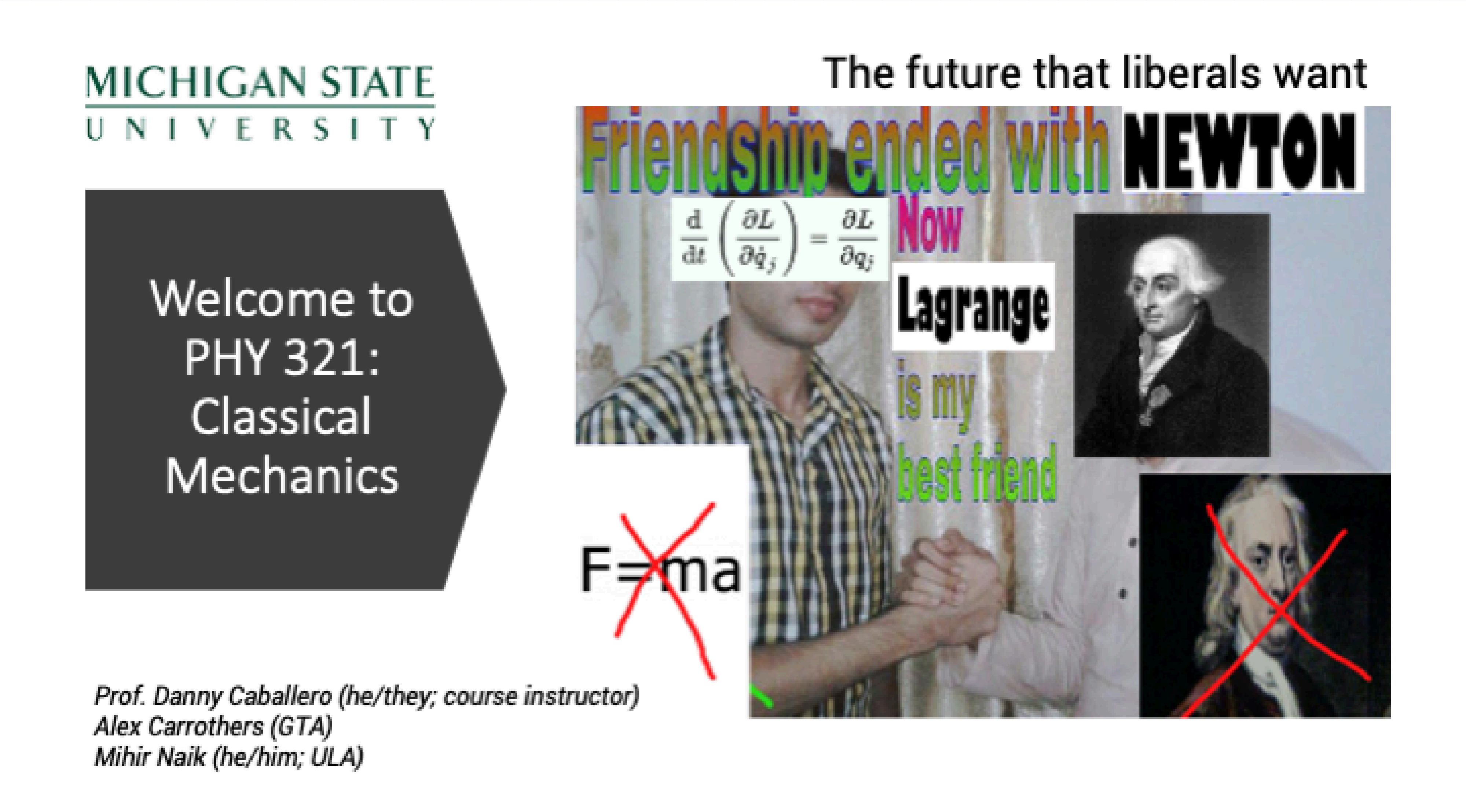
Day 30 - Euler-Lagrange Equation



Seminars this Week

WEDNESDAY, November 5, 2025

Astronomy Seminar, 1:30 pm, 1400 BPS, In Person and Zoom, Host~

Speaker: Nick Konidaris, Carnegie Observatories

Title: The Sephira Project: Astronomical Imaging with Third-Order Intensity Correlations

Seminars this Week

WEDNESDAY, November 5, 2025

FRIB Nuclear Science Seminar, 3:30pm., FRIB 1300 Auditorium and online via Zoom

Speaker: Assistant Professor Xing Wu of The Facility for Rare Isotope Beams (FRIB)

Title: Towards Quantum Control and Sensing of 227ThO Molecules and Other Radioactive Molecules for Nuclear Schiff Moment Search

Please click the link below to join the webinar:

Join Zoom Meeting: https://msu.zoom.us/j/91861947571?pwd=IlUS6RkYdHibaosm4aznsYsctbaMrU.1

Meeting ID: 939 4416 7137

Passcode: 026775

Seminars this Week

THURSDAY, November 6, 2025

Colloquium, 3:30 pm, 1415 BPS, in person and zoom. Host ~ Jay Strader/Laura Chomiuk

Refreshments and social half-hour in BPS 1400 starting at 3 pm

Speaker: Nick Konidaris, Carnegie Observations

Title: SDSS Local Volume Mapper instrument and Early Science Results

Seminars this Week

FRIDAY, November 7, 2025

IReNA Online Seminar, 9:00am, In Person and Zoom, FRIB 2025 Nuclear Conference Room, Light refreshments will be served at 1:50pm.

Hosted by: Sota Kimura (University of Tsukuba)

Speaker Tomoshi Takeda, Hiroshima University, Japan

Title: A New Approach to X-ray Astronomy: Development and Observational Results of the CubeSat Observatory NinjaSat

Seminars this Week

FRIDAY, November 7, 2025

Special HEP Seminar

High Energy Physics Seminar, 1:00 pm, 1400 BPS, Host~ Joey Huston

Speaker: Eric Bachmann, Technische Universität Dresden

Title: Evidence for longitudinal polarization in same-sign WW scattering with the ATLAS detector

Organized by: Joey Huston, Sophie Berkman and Brenda Wenzlick

Seminars this Week

FRIDAY, November 7, 2025

QuIC Seminar, 12:30pm, -1:30pm, 1300 BPS, Virtual only today

Speaker: Philip Crowley, MSU

Title: Quantum dynamics for quantum sensing

Full Scheule is at: https://sites.google.com/msu.edu/quic-seminar/

For more information, reach out to Ryan LaRose

Reminders

We proposed a solution to the line problem that involved an error term $\eta(x)$, which is a small perturbation to the true path $y(x)$. This leads to a perturbed function:

$$Y(x) = y(x) + \alpha\eta(x)$$

where α is a small parameter.

We proposed that there's a functional $f(Y, Y', x)$ that depends on a function $Y(x)$, its derivative $Y'(x)$, and the independent variable x such that:

$$\int_{s_1}^{s_2} f(Y, Y', x)\, dx > \int_{s_1}^{s_2} f(y, y', x)\, dx$$

Reminders

By taking the derivative of the functional with respect to α, we can find the condition for which the functional is stationary (i.e., a minimum or maximum).

$$\frac{d}{d\alpha} \int_{s_1}^{s_2} f(Y, Y', x)\, dx \bigg|_{\alpha=0} = 0$$

This (with a lot of math) led us to the following expression:

$$\int_{s_1}^{s_2} \eta(x) \left[\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) \right] dx = 0$$

Clicker Question 30-1

We completed this derivation with the following mathematical statement:

$$\int_{s_1}^{s_2} \eta(x) \left[\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) \right] dx = 0$$

where $\eta(x)$ is an arbitrary function. What does this imply about the term in square brackets?

1. The term in square brackets must be a pure function of x.
2. The term in square brackets must be a pure function of y.
3. The term in square brackets must be a pure function of y'.
4. The term in square brackets must be zero.
5. The term in square brackets must be a non-zero constant.

Clicker Question 30-2

Returning to the line problem,

$$l = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

here, $f(y, y', x) = \sqrt{1 + (y')^2}$, where $y' = \frac{dy}{dx}$.

Apply the Euler-Lagrange equation to find the expression for the function $f(y, y', x)$ in this case. Write your result to find the expression for the term in square brackets:

$$\frac{d}{dx}[?] = 0$$

Click when you have an answer!

Clicker Question 30-3

With,

$$y' = \pm\sqrt{\frac{c^2}{1+c^2}}$$

where c is a constant, the solution expresses a straight line.

1. True and I can prove it!
2. True, but I'm not sure how to prove it.
3. False, I think this is incorrect.
4. I don't know.

Clicker Question 30-4

We derived the time that it takes to run from a point on the shore to a point in the water, T:

$$T = \frac{1}{v_1}\left(x_1^2 + (y - y_1)^2\right)^{1/2} + \frac{1}{v_2}\left(x_2^2 + (y_2 - y)^2\right)^{1/2}$$

To find the minimal time, what derivative should we take?

1. $\dfrac{dT}{dx}$
2. $\dfrac{dT}{dy}$
3. $\dfrac{dT}{dt}$
4. Something else?

Clicker Question 30-5

For the brachistochrone problem, the ball moves purely under the influence of gravity. Consider that the ball has moved a vertical distance Δy from rest. What is the speed of the ball at this point?

1. $v = gt$
2. $v = 2g\Delta y$
3. $v = \sqrt{2g\Delta y}$
4. I'm not sure, but $< \sqrt{2g\Delta y}$
5. Something else?