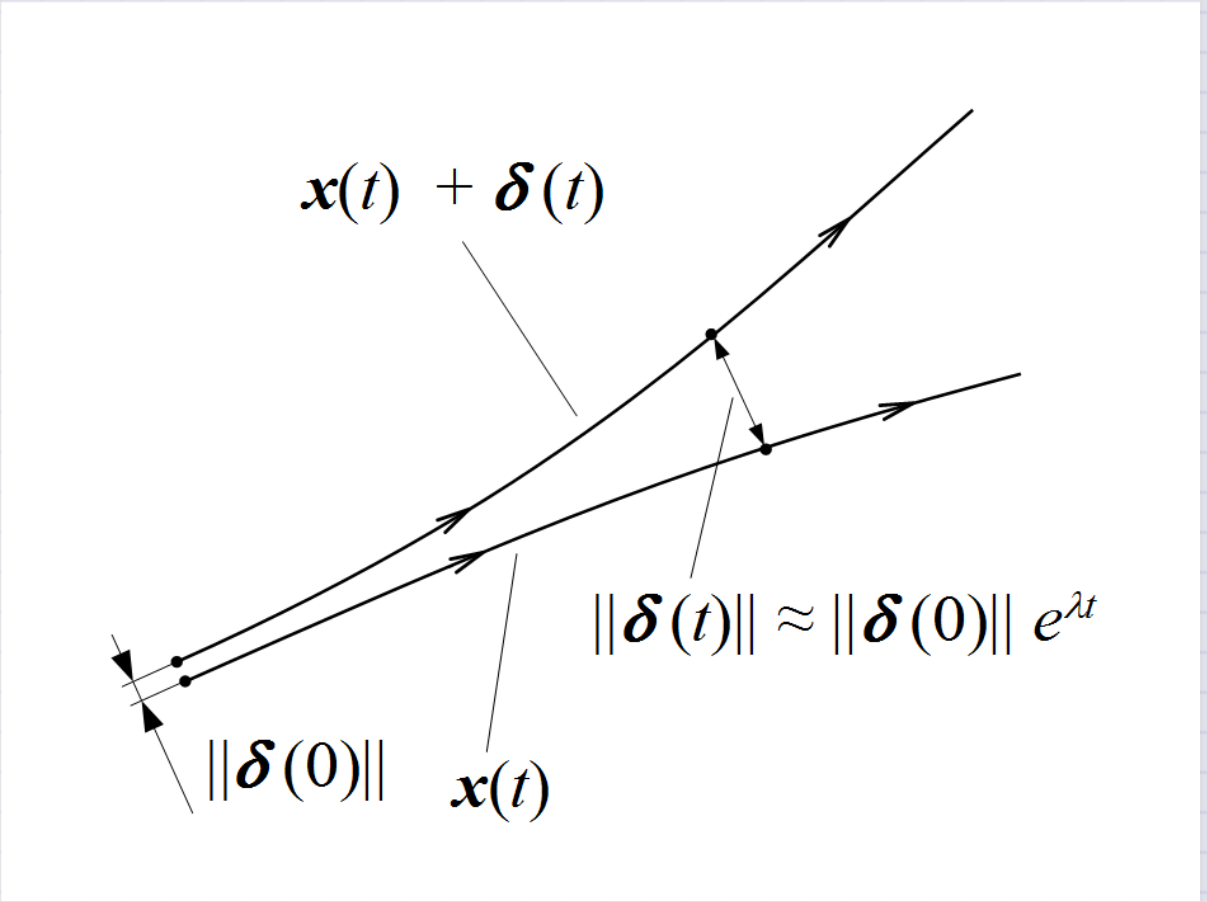
Day 27 - Hallmarks of Chaos

Conceptualizing the Lyapunov Exponent

Trajectories diverge exponentially in time



Announcements

- Midterm 1 is graded
- Homework 7 is due Friday
  - No homework next week
- Midterm 2 will be assigned next Monday (due 14 November)
  - Second project check-in
- **Friday's Class:** We will work HW 7 Exercises 2 & 3 together

Seminars This Week

WEDNESDAY, October 29, 2025

Astronomy Seminar, 1:30 pm, 1400 BPS, In Person and Zoom, Host~

Speaker: Michael Radic, University of Chicago

Title:

Zoom Link: https://msu.zoom.us/j/93334479606?pwd=OtIXPWhRPBfzYu53sI3trSJIaBYI7C.1

Meeting ID: 933 3447 9606

Passcode: 825824

Seminars This Week

WEDNESDAY, October 29, 2025

PER (Physics Education Research Seminar), 3:00 pm., BPS 1400 in person and zoom

Speaker: **Eric Burkholder**, *Assistant Professor at Auburn University*

Title: **Could we make physics more accessible by teaching real physics?**

Zoom Link: https://msu.zoom.us/j/96470703707

Meeting ID: 964 7070 3707

Passcode: PERSeminar

Seminars This Week

WEDNESDAY, October 29, 2025

FRIB Nuclear Science Seminar, 3:30pm., FRIB 1300 Auditorium and online via Zoom

Speaker: Professor Dien Nguyen of the University of Tennessee, Knoxville

Title: The Pairing Mechanism of Short Range Correlations and the impact of Nuclear Structure

Please click the link below to join the webinar:

Join Zoom Meeting: https://msu.zoom.us/j/93944167137?pwd=jzvwvbL8YqDnJNpzDPat8IHcrFdtC5.1

Meeting ID: 939 4416 7137

Passcode: 239049

New Course Alert: CMSE 491 – Quantum Information Science and Engineering

Get started in the emerging field of quantum engineering!

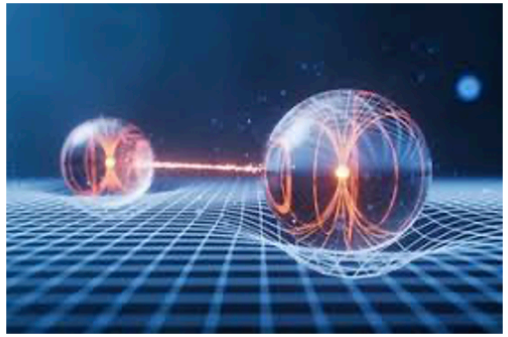
- 🧠 What's it about? Quantum systems, quantum hardware, and real-world applications in computing, networking, and sensing.
- 🕐 When: M/W/F 12:40–1:30 PM
- 📍 Where: Farrall Agricultural Engineering Hall 119
- 👩‍🏫 Instructor: Dr. Sarah Frechette (ERC C107, rober964@msu.edu)

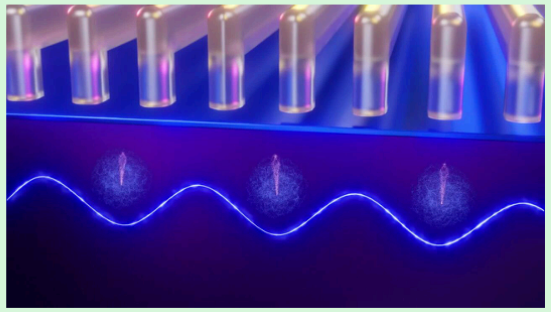
🟦 Who's it for?

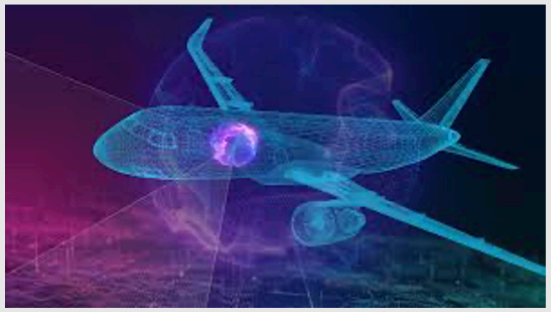
Physics, engineering, and computing majors—or anyone curious about quantum tech.

Quantum Communication

- How can quantum mechanics make communications *unhackable*?
- What does it take to send quantum information over long distances?
- What technologies could form the backbone of a future quantum internet?







CMSE 491:
Quantum Information Science and Engineering

- How do researchers physically build a quantum computer?
- How do we control and measure something that changes when you look at it?

Quantum Computing

- Where might quantum sensing make a difference – in medicine, navigation, or space exploration?
- How do quantum sensors differ from classical sensors?

Quantum Sensing

Hallmarks of a Classically Chaotic System

1. **Deterministic**
2. **Sensitive to Initial Conditions**
3. **Non-periodic Behavior**
4. **Strange Attractors**
5. **Parameter Sensitivity**
6. (Sometimes) **Periodic Behavior**

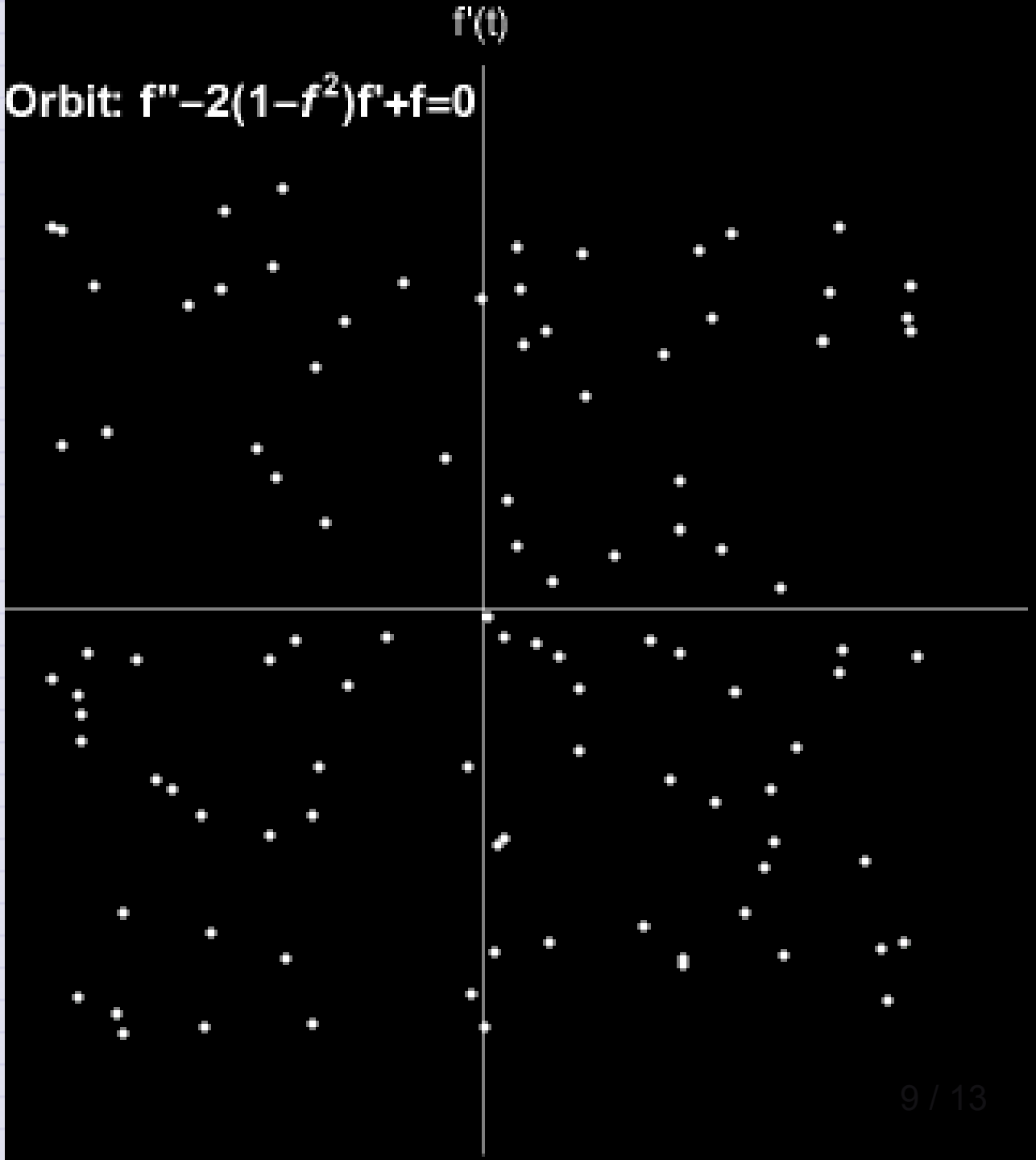
Limit Cycle

A **limit cycle** is a closed trajectory in phase space that is an attractor for a dynamical system.

The **Van der Pol Oscillator** exhibits a limit cycle.

$$\ddot{x} - \mu(1 - x^2)\dot{x} + x = 0$$

Random initial conditions converge to a limit cycle. Modeled with $\mu = 2$.



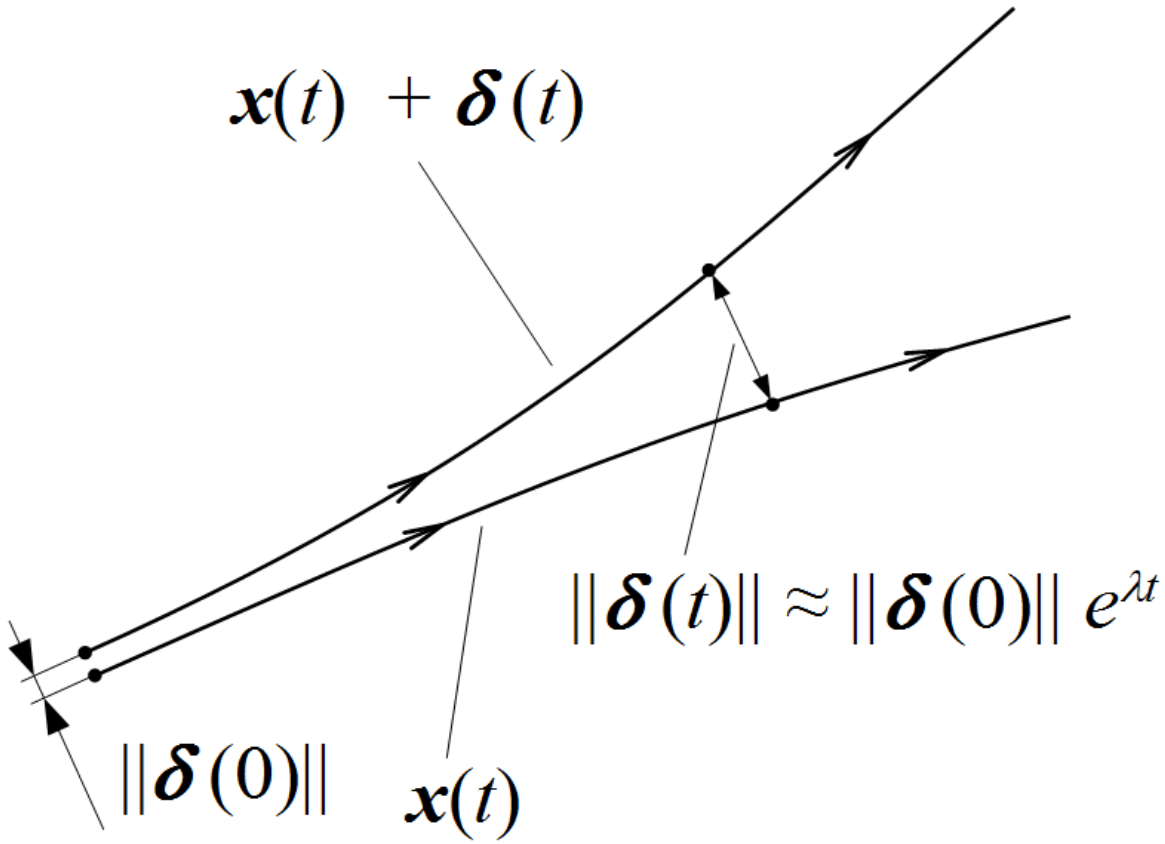
The Lyapunov Exponent

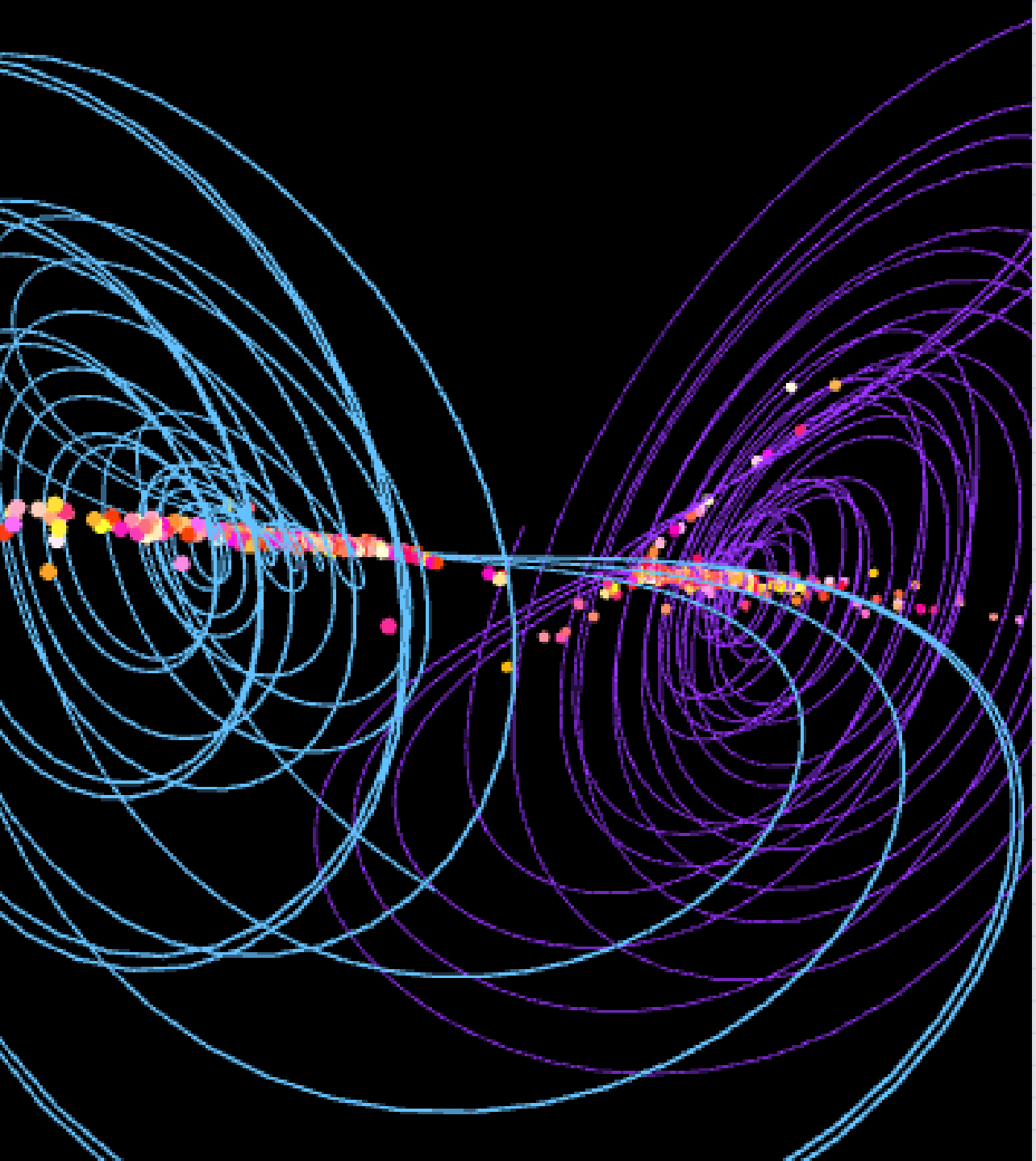
$\vec{\delta}(t)$ is the separation vector between two trajectories in phase space
$\vec{\delta}(t) = \vec{x}_2(t) - \vec{x}_1(t)$.

Do trajectories diverge exponentially in time, $|\vec{\delta}(t)| \approx |\vec{\delta}(0)|e^{\lambda t}$?

Each phase coordinate can change at a different rate:
$\vec{\lambda} = \langle \lambda_1, \lambda_2, \ldots, \lambda_n \rangle$.

Largest $\lambda_i > 0$? Chaotic system.





Strange Attractors

A **strange attractor** is a set of points in phase space that a chaotic system approaches.

Chen Attractor

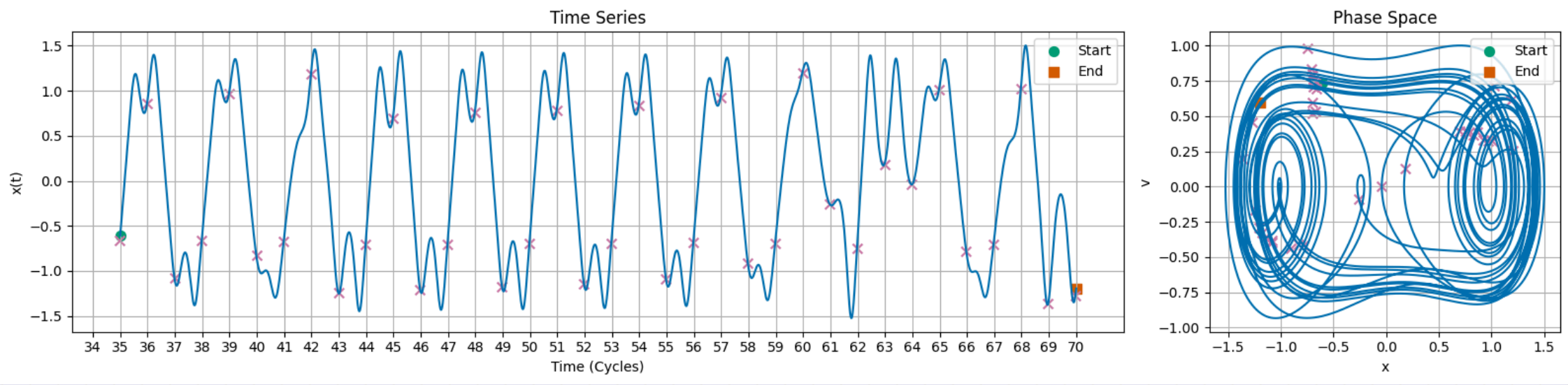
$$\dot{x} = \alpha x - yz$$
$$\dot{y} = \beta y + xz$$
$$\dot{z} = \gamma z + xy/3$$

$\alpha = 5, \beta = -10, \gamma = -0.38.$

Interactive 3D Model

Example 1: Duffing Equation

$$\ddot{x} + \beta\dot{x} + \alpha x + \gamma x^3 = F_0 \cos(\omega t)$$



Exhibits Periodic and Chaotic Behavior

Illustrates period doubling bifurcations as route to chaos

Example 2: Lorenz System

$$\dot{x} = \sigma(y - x)$$
$$\dot{y} = x(\rho - z) - y$$
$$\dot{z} = xy - \beta z$$

Exhibits sensitive dependence on initial conditions

Demonstrates the concept of a strange attractor