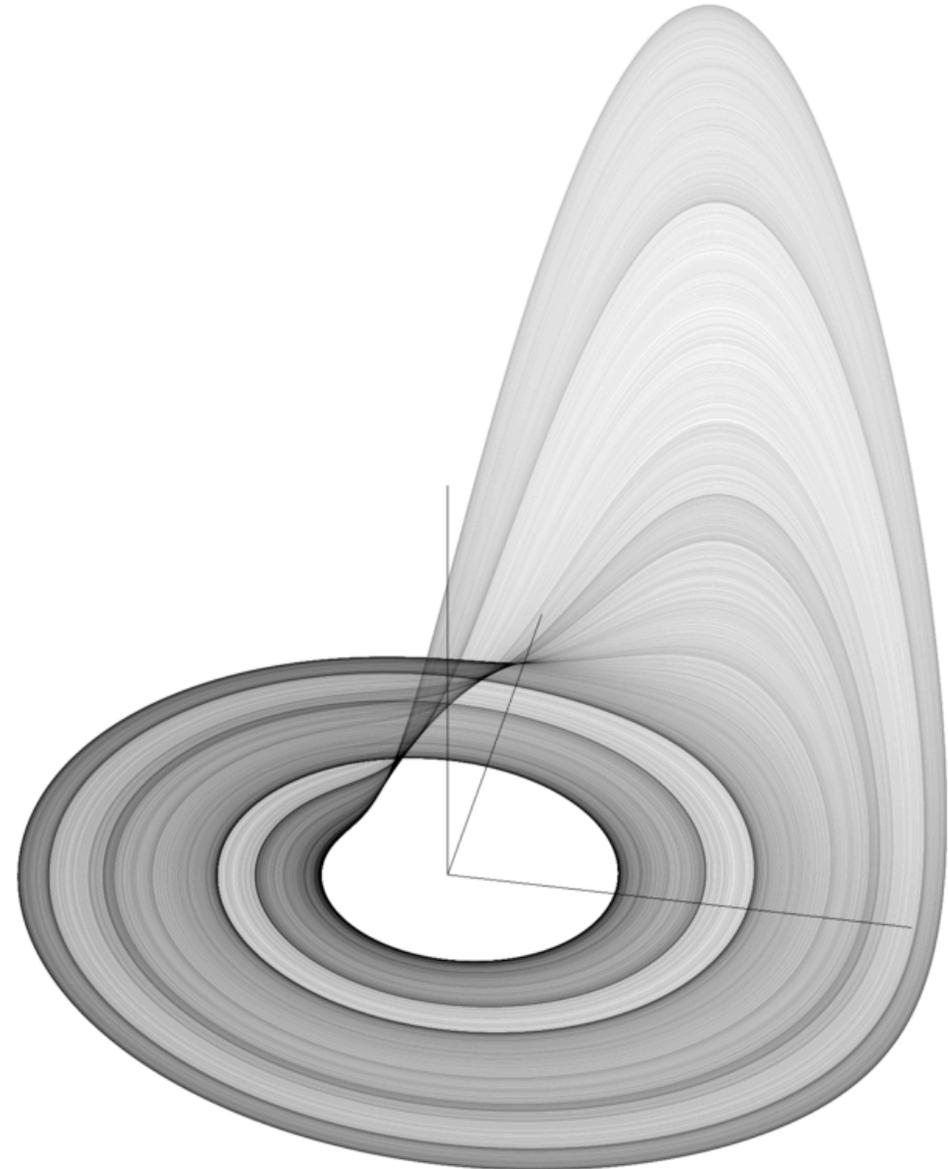


Day 17 - Nonlinear Dynamics

Rössler Attractor

$$\begin{aligned}\dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= b + z(x - c)\end{aligned}$$

There are no crossings of the path shown in this picture



Seminars this week

MONDAY, February 23, 2026

Condensed Matter Physics Seminar, 4:10p.m., BPS 1400 & Zoom

Speaker: Ilija Zeljkovic, Boston College

Title: Atomic-scale imaging of symmetry-broken electronic states in kagome superconductors

Zoom Link: <https://msu.zoom.us/j/93613644939>

Password: CMP

TUESDAY, February 24, 2026

High Energy Physics Seminar, 1:30p.m., BPS 1400 & Zoom

Speaker: Daniel Adamiak/Artemiy Filippov, MSU

Title: Investigating the phenomenon of double descent in cross-validation

*For the Zoom link – Please contact Joey Huston, Sophie Berkman and/or Brenda Wenzlick

WEDNESDAY, February 25, 2026

Astronomy Seminar, 1:30 p.m., BPS 1400 & Zoom

Speaker: Juliette Becker, University of Wisconsin-Madison

Title: TBA

Zoom Link: [https://msu.zoom.us/j/93334479606?](https://msu.zoom.us/j/93334479606?pwd=OtIXPWhRPBfzYu53sl3trSJlaBYI7C.1)

[pwd=OtIXPWhRPBfzYu53sl3trSJlaBYI7C.1](https://msu.zoom.us/j/93334479606?pwd=OtIXPWhRPBfzYu53sl3trSJlaBYI7C.1)

Passcode: 825824

WEDNESDAY, February 25, 2026

FRIB Nuclear Science Seminar, 3:30 p.m., FRIB 1300 & Zoom

Speaker: Jordan Stomps, Oak Ridge National Laboratory

Title: Data Science and Engineering for Nuclear Nonproliferation

Zoom Link: [https://msu.zoom.us/j/93742845358?](https://msu.zoom.us/j/93742845358?pwd=vlen3rIRdk8NHBSOxVIM1Aj2cP144m.1)

[pwd=vlen3rIRdk8NHBSOxVIM1Aj2cP144m.1](https://msu.zoom.us/j/93742845358?pwd=vlen3rIRdk8NHBSOxVIM1Aj2cP144m.1)

Passcode: 416741

THURSDAY, February 26, 2026

Colloquium, Seminar, 3:30 p.m., BPS 1415 & Zoom

Refreshments at 3:00 BPS in BPS 1400

Speaker: Dylan Yost, Colorado State

Title: Precision tests of quantum electrodynamics through hydrogen spectroscopy and vacuum birefringence

Zoom Link: <https://msu.zoom.us/j/94951062663>

Password: 2002

*For more information and scheduling a time to meet with the speaker, please see the calendar: <https://pa.msu.edu/news-events-seminars/colloquium-schedule.aspx>

FRIDAY, February 27, 2026

QuIC, Seminar, 12:40 p.m., BPS 1300 & Zoom

Speaker: Kevin Sung, IBM

Title: Enhancing Chemistry on Quantum Computers with Fermionic Linear Optical Simulation

*For the full schedule, please see: <https://sites.google.com/msu.edu/quic-seminar/> or for more information, please reach out to Ryan LaRosa directly

FRIDAY, February 27, 2026

IReNA Online Seminar, 2:00 pm, Zoom

Light refreshments at 1:50pm in 2025 Nuclear Conference Room - FRIB

Hosted by: Aldana Grichener (University of Arizona & Observatory)

Speaker: Shivani Shah, North Carolina State University

Title: Actinide Abundances, Variation, and Evolution in Metal-Poor Stars

Zoom Link: <https://msu.zoom.us/j/827950260>

Password: CENAM

Reminders: Conservative Forces

- The curl of a conservative force is zero

$$\nabla \times \vec{F}_{cons} = 0$$

- Work done by a conservative force is path-independent

$$\underbrace{\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{cons} \cdot d\vec{r}}_{\text{path a}} = \underbrace{\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{cons} \cdot d\vec{r}}_{\text{path b}}$$

- Work done by a conservative force around a closed path is zero

$$\oint \vec{F}_{cons} \cdot d\vec{r} = 0$$

Reminders: Conservative Forces

- The work done by a conservative force is equal to the negative of the change in potential energy

$$W = -\Delta U$$

- A conservative force can be expressed as the gradient of a scalar function

$$\vec{F}_{cons} = -\nabla U$$

Reminders: Equilibrium (aka Critical or Fixed) Points

We found equilibrium points by setting the derivative of the potential energy to zero:

$$\frac{dU}{dx} = 0$$

We then determined if these points were stable or unstable by looking at the second derivative of the potential energy:

$$\frac{d^2U}{dx^2} > 0 \quad \text{stable}$$
$$\frac{d^2U}{dx^2} < 0 \quad \text{unstable}$$

Relationship to Differential Equations

By setting $\frac{dU}{dx} = 0$, we are finding the equilibrium points where the force is zero,

$$-\frac{dU}{dx} = F_x = 0$$

If we consider the typical form of a differential equation, we can see that we are seeking the points where the differential equation is zero,

$$m\ddot{x} = F(x) \quad m\ddot{x} = 0$$

This approach is a powerful way to understand the behavior of a system. And we can do so geometrically!

Clicker Question 17-1

Let $\dot{x} = \sin x$. Set up the integral that could be used to solve for $t(x)$.

1. $\int \frac{dx}{\sin x}$

2. $\int \frac{dx}{\cos x}$

3. $\int \frac{dx}{\tan x}$

4. $\int \frac{dx}{\cot x}$

5. ???

Clicker Question 17-2

We can integrate this with $x(0) = x_0$ to find $t(x)$:

$$t(x) = \ln(|\csc x - \cot x|) - \ln(|\csc x_0 + \cot x_0|)$$

Find $x(t)$? 😞😞😞

Instead find the equilibrium points (x^*) of the system. n is an integer.

1. $x^* = 0$
2. $x^* = 0, \pm\pi$
3. $x^* = \pm\pi/2$
4. $x^* = n\frac{\pi}{2}$
5. $x^* = n\pi$

Clicker Question 17-3a

Sketch the differential equation $\dot{x} = \sin x$ in the phase space x vs. \dot{x} .

1. Note where the plot crosses the x -axis. These are the critical/equilibrium points, x^* . Identify the critical points.
2. By definition, $\dot{x} > 0$ is a "flow to the right" and $\dot{x} < 0$ is a "flow to the left". Sketch the direction of the flow - this should only appear in the x -axis.

Click when you and your table are done.

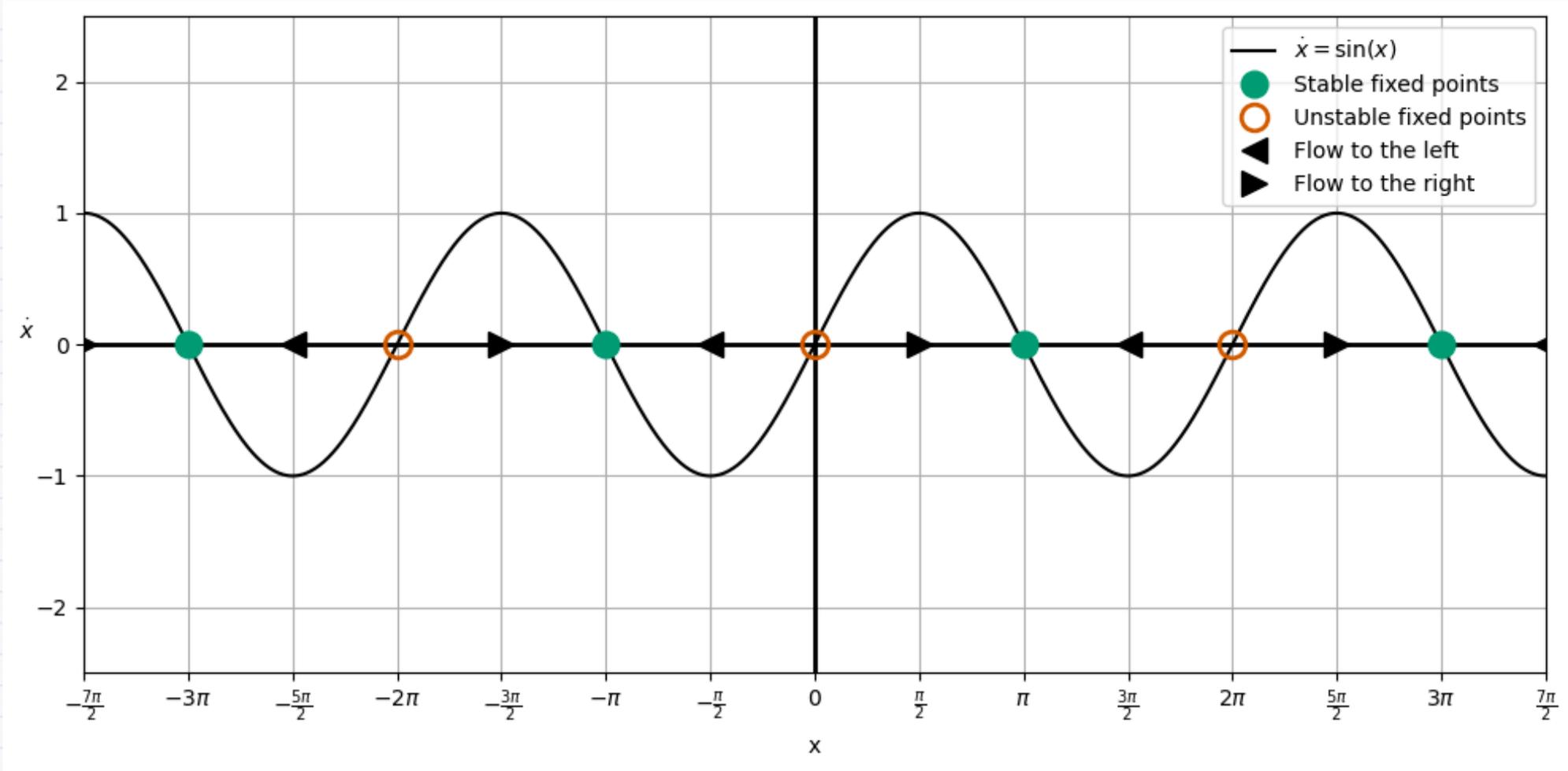
Clicker Question 17-3b

Sketch the differential equation $\dot{x} = \sin x$ in the phase space x vs. \dot{x} .

3. Look at the flow directions and the critical points. What can you say about the stability of the critical points? We use closed circles for stable points and open circles for unstable points. Add these to your plot.

Click when you and your table are done.

Phase Space Diagram for $\dot{x} = \sin x$



Clicker Question 17-4a

Consider now the differential equation $\dot{x} = x^3 - x$. To find $t(x)$, we can integrate:

$$t(x) = \int_{x_0}^x \frac{dx'}{x'^3 - x'}$$

That yields the following solution (🙄🙄🙄):

$$t(x) = \left(\frac{1}{2} \ln(1 - x^2) - \ln(x) \right) - \left(\frac{1}{2} \ln(1 - x_0^2) - \ln(x_0) \right)$$

1. Find the equilibrium points (x^*) of the system.
2. Sketch the differential equation $\dot{x} = x^3 - x$ in the phase space x vs. \dot{x} .

Click when you and your table are done.

Clicker Question 17-4b

Consider now the differential equation $\dot{x} = x^3 - x$. To find $t(x)$, we can integrate:

$$t(x) = \int_{x_0}^x \frac{dx'}{x'^3 - x'}$$

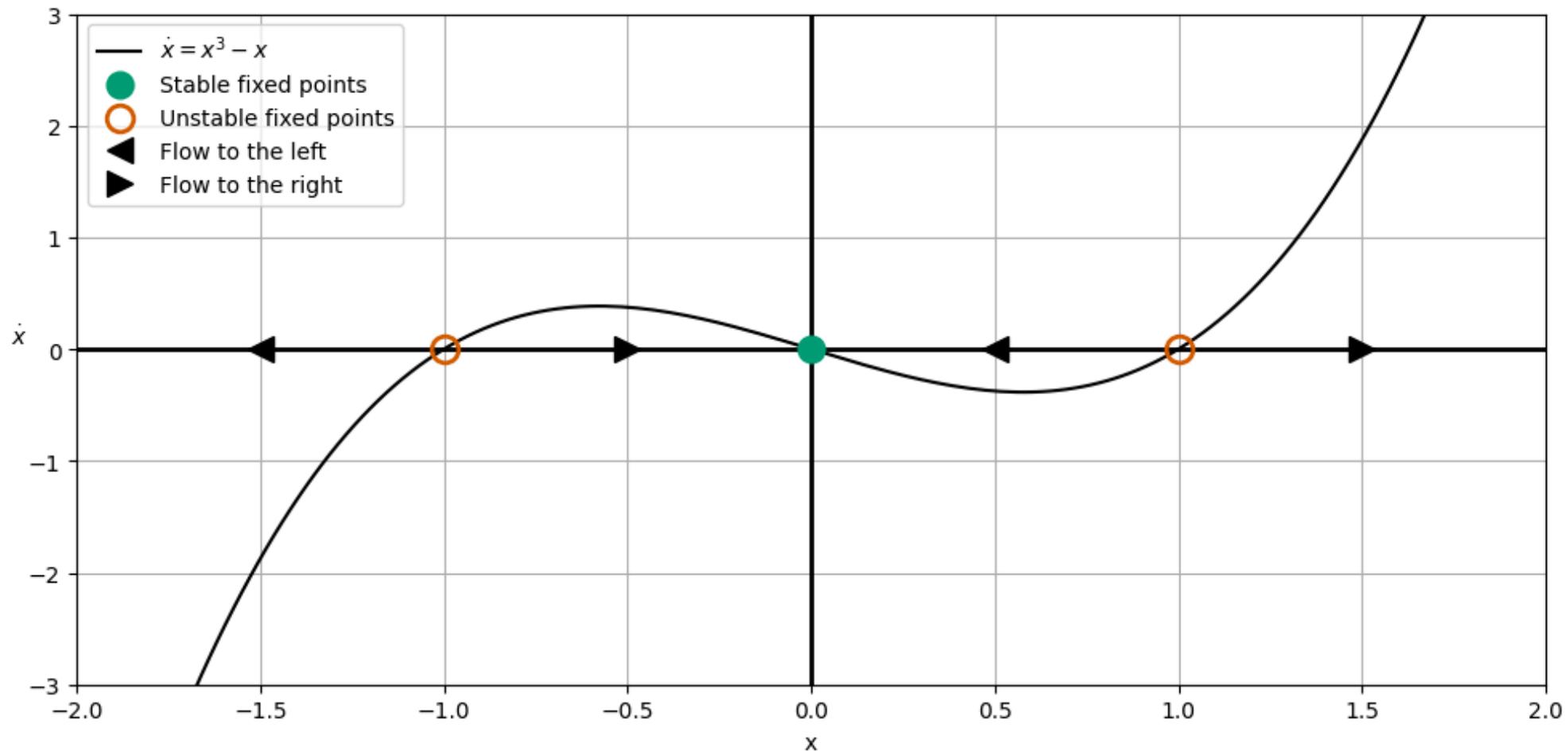
That yields the following solution (🙄🙄🙄):

$$t(x) = \left(\frac{1}{2} \ln(1 - x^2) - \ln(x) \right) - \left(\frac{1}{2} \ln(1 - x_0^2) - \ln(x_0) \right)$$

3. What can you say about the stability of the critical points? Add these to your plot.

Click when you and your table are done.

Phase Space Diagram for $\dot{x} = x^3 - x$



The Harmonic Oscillator Gets a Bad Rap

The SHO is a linear system. It's boring. It's predictable. It's stable. But it can help us understand nonlinear 2nd order ODEs and thus more complex systems.

Consider the physical pendulum. The equation of motion is

$$mL^2\ddot{\theta} = -mgL \sin \theta$$

Or more simply:

$$\ddot{\theta} = -\frac{g}{L} \sin \theta$$

In the case of small angles, $\sin \theta \approx \theta$, and we have a linear system.

$$\ddot{\theta} = -\frac{g}{L} \theta$$

Examples of SHOs

- The spring-mass system $\ddot{x} = -\omega^2 x$
- The simple pendulum $\ddot{\theta} = -\frac{g}{L}\theta$
- The LC circuit $\ddot{q} = -\frac{1}{LC}q$
- Water in a u-tube $\ddot{h} = -\frac{2g\rho A}{M}h$
- A jump rope $\ddot{u} = \frac{T}{\lambda} \left(\frac{n\pi}{d}\right)^2 u$

Any system with a local minimum in the potential energy can be modeled as an SHO.