Day 23 - Homework Session

Announcements

- Homework 3 is graded
- Midterm 1 is still being graded
  - Re: Problem 2 $\rightarrow$ HOLY CRAP YOU ALL ARE AMAZING
- Homework 5 is due tonight
- Homework 6 is due next Friday
- Danny will be out next Wednesday
  - Class will be on zoom (usual link)

Reminders

We solved the damped harmonic oscillator equation:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$$

We chose a solution (**ansatz**) of the form

$$x(t) = C_1 e^{rt} + C_2 e^{rt}$$

and computed the roots of the characteristic equation:

$$r^2 + 2\beta r + \omega_0^2 = 0$$

We found the roots to be:

$$r = -\beta \pm \sqrt{\beta^2 - \omega_0^2}$$

Weak Damping

We found that when $\beta^2 < \omega_0^2$, the roots are complex:

$$r = -\beta \pm i\sqrt{\omega_0^2 - \beta^2}$$

This means that the solution is oscillatory:

$$x(t) = e^{-\beta t}\left(C_1 \cos(\sqrt{\omega_0^2 - \beta^2}t) + C_2 \sin(\sqrt{\omega_0^2 - \beta^2}t)\right)$$

The solution is a damped oscillation with frequency $\omega_1 = \sqrt{\omega_0^2 - \beta^2}$.

Strong Damping

When $\beta^2 > \omega_0^2$, the roots are real:

$$r = -\beta \pm \sqrt{\beta^2 - \omega_0^2}$$

This means that the solution is not oscillatory:

$$x(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$$

where $r_1 = -\beta + \sqrt{\beta^2 - \omega_0^2} < 0$ and $r_2 = -\beta - \sqrt{\beta^2 - \omega_0^2} < 0$.

The solution is the sum of two exponentials with different decay rates.

Critical Damping

When $\beta^2 = \omega_0^2$, the roots are real and equal (repeated roots):

$$r = -\beta$$

This means that the solution is not oscillatory, but also that our ansatz is not sufficient. The correct form of the solution is:

$$x(t) = (C_1 + C_2 t)e^{-\beta t}$$

In most cases, we will work with weak damping.