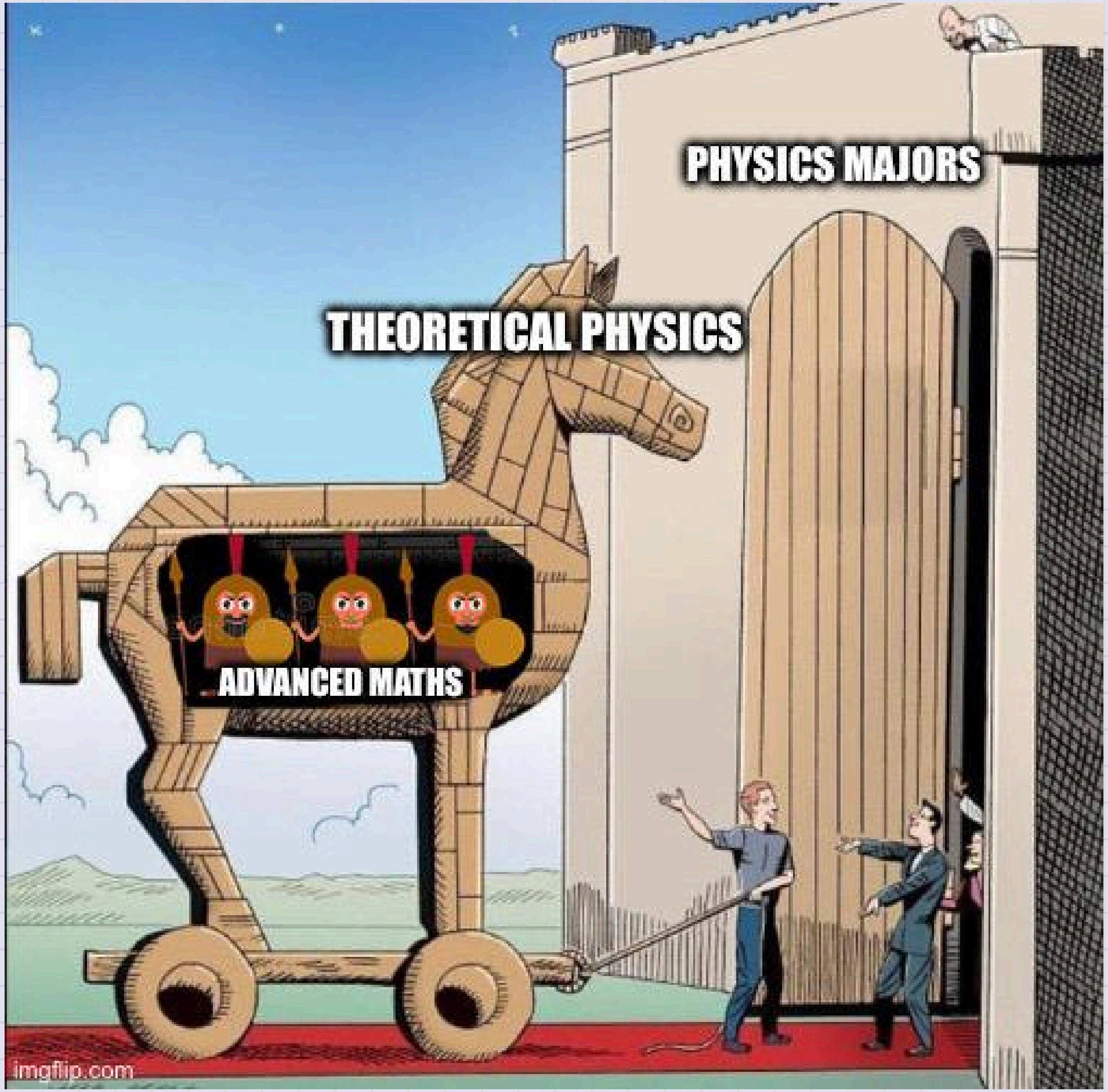
Day 28 - Homework Session



Announcements

- Homework 7 is due Sunday
- Midterm 2 will be posted Monday
- Instructions for final project will be posted next week
    - We have one midterm and one homework assignment left!
- DC's group is looking for a research assistant
    - Our group pays ASMSU recommended living wage: $21/hr
    - Current project has 50+ hours worth of work
    - Max employment: 10 hrs/week

Exercise 2 Where does the energy go?

The damped harmonic oscillator is described by the equation of motion is:

$$m\ddot{x} + b\dot{x} + kx = 0$$

where m is the mass, b is the damping coefficient, and k is the spring constant.

The damping term ($F_{damp} = -b\dot{x}$) models the dissipative forces acting on the oscillator. The total energy for the oscillator is given by the sum of the kinetic and potential energies,

$$E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2.$$

- 2a What is the energy per unit time dissipated by the damping force?

Exercise 2 Where does the energy go?

$$m\ddot{x} + b\dot{x} + kx = 0$$

$$E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$$

- 2b Take the time derivative of the total energy and show that it is equal (in magnitude) to the energy dissipated by the damping force.
- 2c What is the sign relationship between the energy dissipated by the damping force and the time derivative of the total energy?

Exercise 3, Unpacking the critically damped solution

The solution for critical damping ($\beta = \omega_0$) is given by,

$$x(t) = x_1(t) + x_2(t) = Ae^{-\beta t} + Bte^{-\beta t},$$

where A and B are constants.

Notice the second solution $x_2(t) = Bte^{-\omega_0 t}$ has an additional linear term t. We glossed over this solution in class, but it is important to understand why this term is present because it tells us about solving differential equations with pathologically difficult-to-see solutions.

Exercise 3, Unpacking the critically damped solution

Start with the under damped solutions,

$$y_1(t) = e^{-\beta t}\cos(\omega_1 t) \quad \text{and} \quad y_2(t) = e^{-\beta t}\sin(\omega_1 t),$$

where we have used the notation $\omega_1 = \sqrt{\omega_0^2 - \beta^2}$.

- 3a (3pt). Show that you can recover the first solution $x_1(t)$ by taking the limit of $\beta \to \omega_0$ of $y_1(t)$.
- 3b (3pt). Show that you cannot recover the second solution $x_2(t)$ by taking the limit of $\beta \to \omega_0$ of $y_2(t)$ directly. What do you get?
- 3c (4pt). If $\beta \neq \omega_0$, you can divide $y_2(t)$ by ω_1. Now show that in the limit $\beta \to \omega_0$ of $y_2(t)/\omega_1$, we recover the form of $x_2(t)$.