Homework 11 (Due. Mar. 27)

1. Star Wars with relativity

Space Probe #1 passes very close to earth at a time that both we (on earth) and the onboard computer on Probe 1 decide to call t = 0 in our respective frames. The probe moves at a constant speed of $0.5c$ away from earth. When the clock aboard Probe 1 reads t = 60sec, it sends a light signal straight back to earth.

1. At what time was the signal sent, according to the earth's rest frame?
2. At what time in the earth's rest frame do we receive the signal?
3. At what time in Probe 1's rest frame does the signal reach earth?
4. Space Probe #2 passes very close to earth at t = 1sec (earth time), chasing Probe 1. Probe 2 is only moving at 0.3c (as viewed by us). Probe 2 launches a proton beam (which moves at v = 0.21c relative to Probe 2) directed at Probe 1. Does this proton beam strike Probe 1? Please answer twice, once ignoring relativity theory, and then again using Einstein!

2. Muon calculations

The mean lifetime of muons is 2 μs in their rest frame. Muons are produced in the upper atmosphere, as cosmic-ray secondaries.

1. Calculate the mean distance traveled by muons with speed $v = 0.99c$, assuming classical physics (i.e. without special relativity).
2. Under this assumption, what percentage of muons produced at an altitude of 10 km reach the ground, assuming they travel downward at $v = 0.99c$? Careful here, you will have to think about the distribution of lifetimes given that particle decay is a Poisson process.
3. Calculate the mean distance traveled by muons with speed $v = 0.99c$, taking into account special relativity.
4. Under this assumption, what percentage of muons produced at an altitude of 10 km reach the ground, assuming they travel downward at $v = 0.99c$?

3. π Mesons

A group of π mesons (pions) are observed to be traveling at $u' = 0.8c$ in a physics laboratory. The mean lifetime (let's call it λ) for unstable particles undergoing exponential decay is the average time for a group of particles to be reduced to $1/e$ of their original number. We can express this mathematically as: $N_t = N_0 exp(-\Delta t/\lambda)$. We see then that for $\Delta t = \lambda$, $N_t = N_0 exp(-1) = (1/e) \cdot N_0$

1. What is the γ–factor for these pions?
2. If the pions' proper mean lifetime is $\lambda = 2.6 \times 10^{-8}$ s, what is the lifetime (λ') as observed in the laboratory frame?
3. If there were initially 32,000 pions, how many will be left after they have traveled 36 meters, from the source to a detector (as measured in the laboratory frame)?
4. Show that this number is the same, as calculated in the rest frame of the pions. Explain this calculation.
5. What would the answer to part 4 be if there were no time dilation (i.e., $\Delta\tau = \Delta t'$)?
6. How long does the distance of 36 meters in the laboratory frame appear to the pions in their rest frame?

7. Use the result from part 6 and the proper time elapsed (as calculated in the rest frame of the pions) to find the velocity of the laboratory frame, as calculated in the rest frame of the pions.