

Let's go back to the 2D problem,

$$\nabla^2 V(x, y) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

We can develop an approximate form for this PDE, by approximating the derivative,

$$\frac{\partial^2 V}{\partial x^2} = \frac{V(x+a, y) - 2V(x, y) + V(x-a, y)}{a^2}$$

$$\frac{\partial^2 V}{\partial y^2} = \frac{V(x, y+a) - 2V(x, y) + V(x, y-a)}{a^2}$$

so that,

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = \frac{V(x+a, y) + V(x, y+a) + V(x-a, y) + V(x, y-a) - 4V(x, y)}{a^2}$$

B/c  $\nabla^2 V(x, y) = 0$ ,

$$V(x, y) = \frac{1}{4} [V(x+a, y) + V(x, y+a) + V(x-a, y) + V(x, y-a)]$$

$V @ x, y$  is average of surrounding pts!

