PHY 481
Fall 2018
November 7, 2018

Name: ______________________________
Exam #2
Time Limit: 120 minutes

Answer the questions in the spaces provided on the question sheets, **making sure to include units**. If you run out of room for an answer, continue on the pages marked **Extra Work**, but indicate that you have done so.

Your answers should include explanations where necessary (or requested) as well as appropriate units and labels (as needed). Write legibly – If we can't read it, we can't grade it. If you have a question, ask your instructor not your classmate.

Problem	Points	Score
1	32	
2	27	
3	21	
4	20	
5	0	
Total:	100	

This exam is to be completed alone with the aid of a single 8.5X11 sheet of paper with your own notes. We have also provided a formula sheet for you.

By signing below, you are agreeing that you have not received unauthorized assistance during this exam, which includes but is not limited to additional crib sheets & note cards, textbooks, course notes, and/or other stored formulas.

Signature: ____

What song should Danny listen to while grading your exam? ____________________

Spartan Academic Pledge

As a Spartan, I will strive to uphold values of the highest ethical standard. I will practice honesty in my work, foster honesty in my peers, and take pride in knowing that honor is worth more than grades. I will carry these values beyond my time as a student at Michigan State University, continuing the endeavor to build personal integrity in all that I do.

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Useful formulas:

$$\nabla \cdot \frac{\hat{r}}{r^2} = 4\pi\delta(\mathbf{r}) \qquad \nabla \cdot \frac{\hat{r}}{r} = \frac{1}{r^2} \qquad \nabla \cdot (f\mathbf{A}) = \nabla f \cdot \mathbf{A} + f\nabla \cdot \mathbf{A}$$

$$f(x) \approx f(x_0) + f'(x_0)(x-x_0) + \frac{1}{2}f''(x_0)(x-x_0)^2 + \ldots \qquad f(x) \approx f(0) + f'(0)(x) + \frac{1}{2}f''(0)(x)^2 + \ldots$$

$$(1 \pm \epsilon)^n \approx 1 \pm n\epsilon + \ldots \qquad \sin\epsilon \approx \epsilon - \ldots \qquad \cos\epsilon \approx 1 - \epsilon^2/2 + \ldots \qquad \ln(1+\epsilon) \approx \epsilon - \ldots$$

$$\int \frac{dx}{x} = \ln(x) + C \qquad \int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \log\left(\sqrt{x^2+a^2} + x\right) + C \qquad \int \frac{x}{\sqrt{x^2+a^2}}dx = \sqrt{x^2+a^2} + C$$

$$\int \frac{1}{(x^2+a^2)^{3/2}}dx = \frac{x}{a^2\sqrt{x^2+a^2}} + C \qquad \int \frac{x}{(x^2+a^2)^{3/2}}dx = -\frac{1}{\sqrt{x^2+a^2}} + C$$

$$\frac{d}{dx}(\sin(ax)) = a\cos(ax) \qquad \frac{d}{dx}(\cos(ax)) = -a\sin(ax)$$

$$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) \qquad \int \cos(ax)\, dx = \frac{1}{a}\sin(ax)$$

$$V(r,\theta) = \sum_l \left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l(\cos\theta) \qquad P_0(\cos\theta) = 1 \qquad P_1(\cos\theta) = \cos\theta \qquad P_2(\cos\theta) = \frac{1}{2}\left(3\cos^2\theta - 1\right)$$

$$E_{out}^{\perp} - E_{in}^{\perp} = \frac{\sigma}{\varepsilon_0} \qquad \left.\frac{\partial V_{out}}{\partial n} - \frac{\partial V_{in}}{\partial n}\right|_S = -\frac{\sigma}{\varepsilon_0}$$

$$\sigma_B = \mathbf{P} \cdot \mathbf{n}|_S \qquad \rho_B = -\nabla \cdot \mathbf{P}$$

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1. A spherical shell of radius, R, has a known voltage at its surface:

$$V(R,\theta) = V_0\left(4 - 6\cos^2\theta\right)$$

There are no charges outside the shell, and we assume that $V \to 0$ as $r \to \infty$.

(a) (4 points) Write the boundary condition above in terms of the Legendre polynomials, i.e., using $P_l(\cos\theta)$.

(b) (8 points) Determine the electric potential **inside** the shell. *You do not need to re-derive the general solution, just use it.*

(c) (8 points) Determine the electric potential **outside** the shell. *You do not need to re-derive the general solution, just use it.*

(d) (8 points) Find the surface charge density $\sigma(\theta)$ on the shell that gives rise to these potentials. Briefly explain your reasoning process here. *If you didn't solve some part of a through c, just explain what you would do.*

(e) (4 points) Sketch a graph of the surface charge density as a function of θ.

2. A charge distribution has 3 point charges: $+q$ located at $\langle d/2, 0, 0\rangle$, another $+q$ located at $\langle -d/2, 0, 0\rangle$, and a $-2q$ located at $\langle 0, 0, d\rangle$.

   (a) (3 points) Sketch the distribution below.

   (b) (3 points) What is Q_{tot} for this distribution?

   (c) (4 points) What is the dipole moment, $\mathbf{p}$, of this distribution (including the direction)?

(d) (8 points) If you get far away, what is the simplest (non-zero) approximation for the potential $V(r, \theta)$? To get full credit, you need to specify the power of r, the θ dependence (if any), and the constant in front in terms of only **given** quantities. (*You do NOT have to derive any formulas from scratch, you can just write down the answer.*)

(e) (5 points) In what region of space (if any) is your expression (above) valid? Briefly, why?

(f) (4 points) Can the origin be moved so the dipole moment vanishes? If so, what would be the leading order potential now (i.e., how would it drop off as an inverse power of r)? If not, why not?

3. A neutral cube centered at the origin with side length b_0 has a "frozen-in" polarization:

$$\mathbf{P} = P_0 \cos\left(\frac{\pi x}{b_0}\right)\hat{\mathbf{z}}$$

(a) (3 points) For a slice of the cube across the $x - y$ plane, sketch the polarization vectors. Does this sketch look any different for parallel slices (i.e., planes parallel to the $x - y$ plane)?

(b) (7 points) Determine the bound surface charge density on each of the six faces of the cube, σ_b.

(c) (3 points) Determine the charge density bound in the volume of the cube, ρ_b.

(d) (8 points) Determine the total bound charge Q_{tot}. Comment on your result, why does it make sense?

4. Consider a two large parallel, metal plates separated by a distance d. One is grounded and the other is held at V_0.

   (a) (10 points) Far from the edges, find the potential between the plates. Start from Laplace's equation and show each step to get full credit.

(b) (10 points) Consider solving this problem computationally (it's a 1D problem, right?). Explain the steps needed to tell the computer to solve the problem using a relaxation method. You may use bullet points and arrows, boxes and arrows, or any other form to get the point across. *You don't have to write the code, just develop the "pseudocode" that could be used to solve it.*

5. **BONUS QUESTION**
   Consider a sphere (radius R) centered on the origin. The electric potential on the surface of the sphere is set to V_0.

   (a) (3 points (bonus)) What is the electric potential at the origin?

   (b) (3 points (bonus)) How do you know your answer to part (a) is correct? *Think about the properties of the solutions to Laplace's equation.*

   (c) (4 points (bonus)) What is the potential everywhere inside the sphere? And how do you know?

   (d) (3 points (bonus)) What is the electric potential outside the sphere? And how do you know?

Extra Work (please indicate which part of the exam you are working)