PHY 481
Fall 2018
October 3, 2018

Name: ______________________________
Exam #1
Time Limit: 120 minutes

Answer the questions in the spaces provided on the question sheets, **making sure to include units**. If you run out of room for an answer, continue on the pages marked **Extra Work**, but indicate that you have done so.

Your answers should include explanations where necessary (or requested) as well as appropriate units and labels (as needed). Write legibly – If we can't read it, we can't grade it. If you have a question, ask your instructor not your classmate.

This quiz is to be completed alone with the aid of a single 8.5X11 sheet of paper with your own notes. We have also provided a formula sheet for you.

By signing below, you are agreeing that you have not received unauthorized assistance during this exam, which includes but is not limited to additional crib sheets & note cards, textbooks, course notes, and/or other stored formulas.

Problem	Points	Score
1	10	
2	15	
3	25	
4	25	
5	25	
Total:	100	

Spartan Academic Pledge

As a Spartan, I will strive to uphold values of the highest ethical standard. I will practice honesty in my work, foster honesty in my peers, and take pride in knowing that honor is worth more than grades. I will carry these values beyond my time as a student at Michigan State University, continuing the endeavor to build personal integrity in all that I do.

Signature: ______________________________

Which of the Harry Potter houses would you be sorted into? ______________________________

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Useful formulas:

$$\nabla \cdot \frac{\hat{r}}{r^2} = 4\pi\delta(\mathbf{r}) \qquad \nabla \cdot \tfrac{\hat{r}}{r} = \frac{1}{r^2} \qquad \nabla \cdot (f\mathbf{A}) = \nabla f \cdot \mathbf{A} + f\nabla \cdot \mathbf{A}$$

$$f(x) \approx f(x_0)+f'(x_0)(x-x_0)+\frac{1}{2}f''(x_0)(x-x_0)^2+\ldots \qquad f(x) \approx f(0)+f'(0)(x)+\frac{1}{2}f''(0)(x)^2+\ldots$$

$$(1 \pm \epsilon)^n \approx 1 \pm n\epsilon + \ldots \qquad \sin\epsilon \approx \epsilon - \ldots \qquad \cos\epsilon \approx 1 - \epsilon^2/2 + \ldots \qquad \ln(1+\epsilon) \approx \epsilon - \ldots$$

$$\ln(a) + \ln(b) = ln(ab) \qquad \ln(a) - \ln(b) = \ln(a/b)$$

$$\int \frac{dx}{x} = \ln(x) + C \qquad \int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \log\left(\sqrt{x^2+a^2}+x\right) + C \qquad \int \frac{x}{\sqrt{x^2+a^2}}dx = \sqrt{x^2+a^2} + C$$

$$\int \frac{1}{(x^2+a^2)^{3/2}}dx = \frac{x}{a^2\sqrt{x^2+a^2}} + C \qquad \int \frac{x}{(x^2+a^2)^{3/2}}dx = -\frac{1}{\sqrt{x^2+a^2}} + C$$

$$\int \cos x\, dx = sinx + C \qquad \int \sin x\, dx = -cosx + C \qquad \frac{d}{dx}\cos x = -\sin x \qquad \frac{d}{dx}\sin x = \cos x$$

$$C_{circle} = 2\pi r \qquad A_{circle} = \pi r^2 \qquad SA_{sphere} = 4\pi r^2$$

If you need some other integral, just ask!

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1. For each statement, circle True or False. You do not need to explain your reasoning.
   (a) (2 points) An electric dipole (one $+q$ and one $-q$) enclosed by any Gaussian surface will have zero electric flux through the Gaussian surface. True False
   (b) (2 points) If the electric potential at some point in space is zero, then the electric field at that same point will be zero. True False
   (c) (2 points) We can use Gauss' Law to determine the electric field of an electric dipole (one $+q$ and one $-q$) at any point in space. True False
   (d) (2 points) The electric potential must be a continuous scalar function, but its derivative can be discontinuous. True False
   (e) (2 points) For the expression $\rho(\mathbf{r}) = c\,\delta^3(\mathbf{r}-\mathbf{a})$ to make physical sense, the units of c need to be charge (or Coulombs in SI). True False

2. Now here's a few questions about delta functions. Make sure your sketches are clear (and label anything you think is important or unclear!)

   (a) (10 points) Sketch the charge distribution $\rho(s, \theta, z) = k\delta(s - a)$. Think about which coordinate system is being used. Explain where the charge is located.

   (b) (5 points) What are the units of k?

3. You are writing a piece of Python code to model the electric field of a charged rod. In this problem, you will discuss the algorithm and write the statements necessary to model the field.

   (a) (10 points) In your own words or with diagrams/flowcharts, describe the algorithm that calculates the electric field of an extended object at some given location. *Make sure there's enough detail in your description that it could be followed easily.*

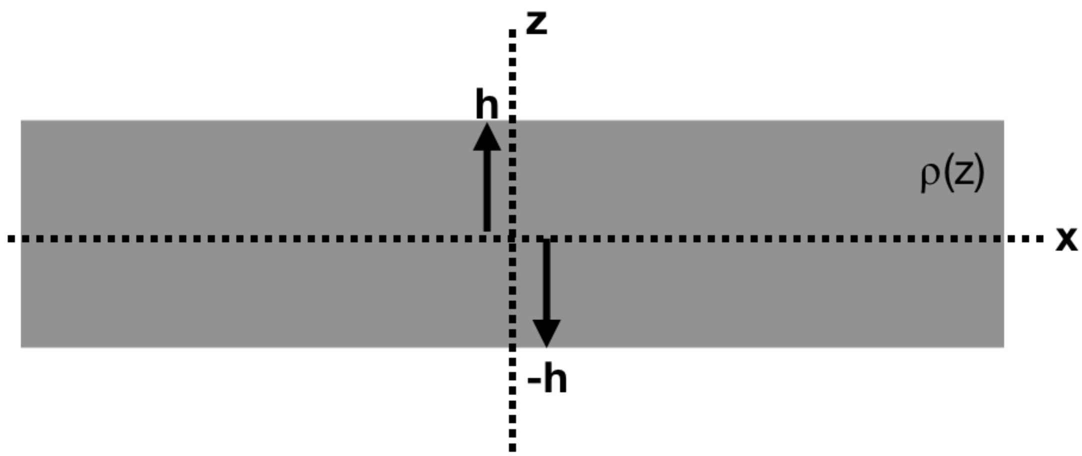
   (b) (5 points) How would you modify the algorithm you produced in part (a) to calculate the field at any given set of locations? You may use additional diagrams and flowcharts if that's useful.

(c) (10 points) Below is a block of code that could be used to implement the algorithm you discussed in part (a). Fill in the missing steps to complete the code.

If you cannot remember specific syntax, that's ok.

```
k = 9e9             ## Permittivity of free space in SI units
Q = 0.1e-6          ## Total charge distributed over the rod in Coulombs
L = 1               ## Length of the rod in meters

N = 4               ## Number of chunks the rod gets broken into
dq = Q/N            ## Charge of a given chunk depends on the number of chunks

Enet = vector(0,0,0)        ## Initialize the electric field
rObs = vector(0.1,0,0)      ## Set the observation location

rod = cylinder(pos=vector(0,L/2,0), axis=vector(0,L,0))

## Create a list of chunk positions
## that is spaced evenly over the rod
listOfChunks = []
for i in range(0,N):

    chunkLoc = vector(0,-L/2+(i+1/2)*L/N,0)
    listOfChunks.append(chunkLoc)
    sphere(pos=chunkLoc)

## Calculation loop that determines the contribution of
## each charge to the total electric field,
## adding that contribution to the total

for thisChunk in listOfChunks:

    ### Enter Your Physics Calculations Here for dE and Enet
```

4. Consider a thick plate of charge centered on the $x - y$ plane with a thickness of $2h$. The plane can be considered infinite in extent in the x-y plane and is centered such that the plane extends between $-h$ and $+h$ in the z-direction (see below).



The charge distribution inside the thick plate varies with height: $\rho(z) = +\rho_0 \cos\left(\frac{\pi z}{2h}\right)$.

(a) (3 points) Sketch a graph of the charge distribution (ρ vs z). Make sure to show what happens for $|z| > h$. Make sure to label your graph.

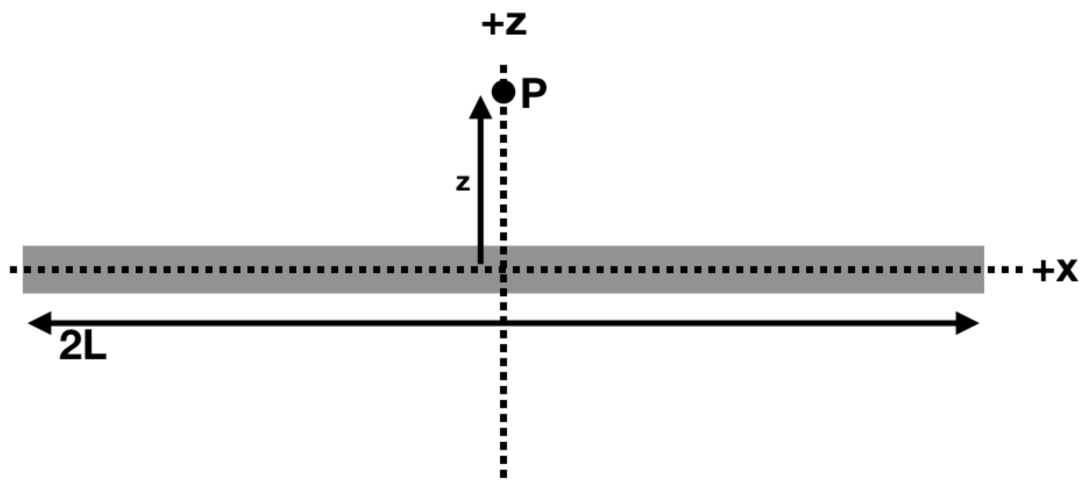
(b) (5 points) Find the magnitude of the electric field outside the plate ($|z| > h$). Which way does the electric field point?

(c) (8 points) Find the electric field inside the plate ($|z| < h$).

(d) (7 points) Sketch the z component of the electric field as a function fo distance from the origin. Make sure to point out any important features in the sketch and label your axes. *Your graph is correct if it is consistent with parts (b) and (c).*

(e) (2 points) Did you use Coulomb's Law or Gauss's Law to solve parts (b) and (c)? Why?

5. Consider a thin rod of length $2L$ that has a total charge of Q. It is located along the x-axis and centered at the origin (shown below). In this problem you will explore the electric potential associated with this rod at point P a location z above the center of the rod.



(a) (6 points) Draw the appropriate vectors ($\mathbf{r}$, $\mathbf{r}'$, and $\boldsymbol{\mathfrak{r}}$) on the diagram above to indicate how you will set up the electric potential calculation. Explain how you know which vector is which.

(b) (9 points) Set up the necessary integral to calculate the electric potential at point P. **(You do not need to solve this integral, but show that you can set it up in a completely integrable form.)**

(c) (4 points) If you calculated this integral, you would find that the result is:

$$V(z) = \frac{Q/2L}{4\pi\varepsilon_0}\left[\log(\sqrt{L^2+z^2}+L) - \log(\sqrt{L^2+z^2}-L)\right]$$

Show that the units of this equation make sense. Notice there are at least two unit checks to do.

(d) (6 points) What limiting behavior do you expect from this equation as $z \to \infty$? Don't say it goes to zero, how should it go to zero? What is the "small parameter"?

(e) (10 points (bonus)) Check part (d) explicitly by taking the appropriate limit. Remember that you need to decide on the correct "small parameter." Does your result agree with your expectation?

(f) (5 points (bonus)) Take the gradient of the equation in part (c) to find the electric field ($\mathbf{E} = -\nabla V$).

Extra Work (please indicate which part of the quiz you are working)